Vanishing viscosity solutions to systems of conservation laws

ABSTRACT: Hyperbolic systems of conservation laws of the form

$$u_t + f(u)_x = 0 \tag{1}$$

admit a Lipschitz semigroup of entropy weak solutions, within a domain of small BV functions. A natural conjecture is that the trajectories of this semigroup are precisely the limits of solutions to the parabolic system

$$u_t + A(u)u_x = \varepsilon u_{xx} \qquad (A = Df) \tag{2}$$

as the viscosity coefficient $\varepsilon \to 0$.

In the literature, the main convergence results for vanishing viscosity approximations rely on the method of compensated compactness, introduced by Tartar and DiPerna. We remark that, while compensated compactness yields the existence of a weak L^∞ solution, all the recent uniqueness results apply only to entropy weak solutions with bounded variation. In order to show that solutions of (2) converge to semigroup trajectories, the key step is thus to prove a-priori bounds on the total variation, uniformly valid as $\varepsilon \to 0$. The talk will address this problem and present some recent progress in this direction.