The self-adjoint operator $H$ in $L^2(\mathbb{R}^d)$ describes a point interaction at the point $a$ if $Hf = -\Delta f$ for every smooth function $f$ with compact support away from $a$. The authors obtain fairly general results on the approximation in the norm resolvent sense of certain point-interaction-Hamiltonians (so-called $\delta$-interaction-Hamiltonians) by regular Schrödinger operators. Moreover, they derive explicit formulas for the resolvent, the eigenvalues, the eigenfunctions, the resonances, the scattering lengths, and the essential, the absolutely continuous and the singular continuous spectra of $\delta$-interaction-Hamiltonians and so-called $\delta'$-interaction-Hamiltonians. These results are then extended to point interactions at finite sets and at lattices; random point-interaction-Hamiltonians are discussed in detail, too. The authors also describe applications of the mathematical theory in a wide variety of models in quantum mechanics.


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- 81U15 Exactly and quasi-solvable systems