

## **Bayesian closed skew Gaussian inversion of seismic AVO data into elastic material properties**

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### **Abstract**

Bayesian inversion is frequently used for seismic inversion. The elastic material properties are assigned a prior probability model and the seismic AVO data is linked to these properties through the likelihood model. This defines the posterior probability model for the elastic properties given the seismic AVO data. Usually, the prior model is log-Gaussian. The log-transform is introduced to make the relation in the likelihood model linear. In this study we introduce a log-closed-skew-Gaussian prior model which generalizes the Gaussian model into skew distributions. The posterior model is analytically tractable also for this case and the posterior pdf may be skew. The Bayesian closed-skew-Gaussian inversion is evaluated on real elastic properties from a North Sea well with associated synthetic seismic AVO data. The results indicates that the mean square error of the predictions can be reduced by approximated a factor of two by introducing skewness in the prior model.

### **Introduction**

Bayesian inversion is frequently used in inversion of seismic AVO data into elastic material properties. In Buland and Omre (2003) the prior model for the logtransformed elastic properties is assumed to be Gaussian while the likelihood model for the seismic data is Gaussian Gauss-linear. Under these assumption the posterior model is Gaussian and analytically tractable. This model is used more out of mathematical convenience than for sound physical reasons. The distributions of logtransformed elastic properties frequently appear as clearly skewed.

In Gonzalez-Farias et al. (2004) a family of multivariate pdfs termed closed skew normal is defined. This family of pdfs is a generalization of the Gaussian pdf into skew distributions. The family is closed under conditioning which is an important property for inversion problems. We draw on results in Gonzalez-Farias et al. (2004) in order to define Bayesian closed skew (CS) Gaussian inversion.

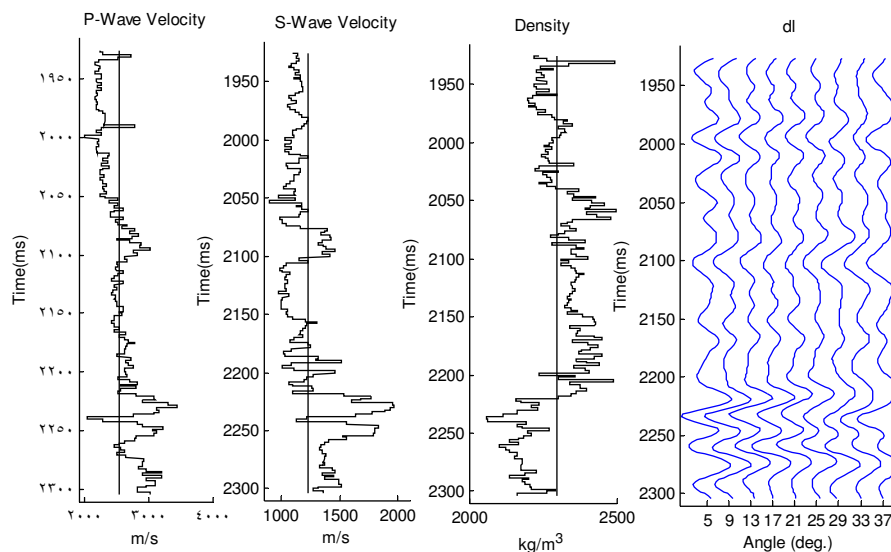
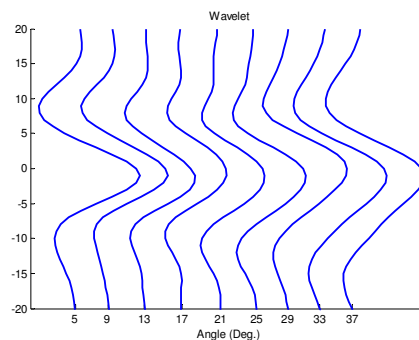


Figure 1: Elastic material property observations and synthetic seismic AVO data.

### Seismic Inversion

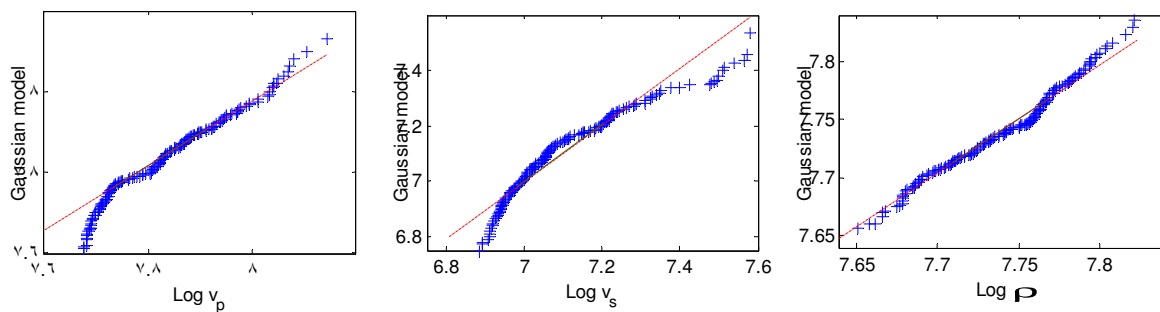
P-wave velocity, S-wave velocity and density is available from well logs in a well in a North Sea sandstone reservoir, see Figure 1. The elastic properties are sampled in 2 msec intervals and are represented by  $\mathbf{m} = (m_i = (\ln v_{pi}, \ln v_{si}, \ln \rho_i); i = 1, \dots, 90)$ . The log-transform is used in order to have a linear relation between by  $\mathbf{m}$  and the seismic AVO data  $\mathbf{d}$ . The seismic AVO data is represented by a convolutional linearized Zoeppritz relation  $\mathbf{d} = \mathbf{WADm} + \mathbf{e}$ , see Buland and Omre (2003). The  $\mathbf{W}$  is a wavelet matrix defined from the wavelet function in Figure 2. The matrix  $\mathbf{A}$  contains the linearized Zoeppritz coefficients, while  $\mathbf{D}$  is a differentiation matrix. The error term  $\mathbf{e}$  is a centered, wavelet-colored Gaussian variable. The study is based on synthetic seismic AVO data in nine angles;  $\mathbf{d}$ , which are generated from the real elastic properties by using relation  $\mathbf{d} = \mathbf{WADm} + \mathbf{e}$ , see Figure 1.



**Figure 2:** Angle dependent wavelet functions.

The objective of the study is to predict the elastic material properties based on the synthetic seismic AVO data. The quality of the predictions will be evaluated by comparing them to the real elastic properties.

Bayesian inversion is normally used in seismic inversion settings like this, see Buland and Omre (2003). The elastic properties are assigned a prior probability model  $p(\mathbf{m})$  and the seismic AVO data are represented by a likelihood model  $p(\mathbf{d} | \mathbf{m})$ . The ultimate solution in Bayesian inversion is the posterior probability model  $p(\mathbf{m} | \mathbf{d})$ . From this model predictions with associated prediction intervals can be inferred. In Buland and Omre (2003) a Bayesian Gaussian inversion approach is used. The prior model for elastic properties  $p(\mathbf{m})$  is assumed to be Gaussian and the likelihood model for seismic AVO data is Gauss-linear. The prior model  $p(\mathbf{m})$  is inferred from observations of elastic properties in the well. The Gaussian assumption is used since it provides analytical tractability of the inversion not because it represents the elastic properties data particularly well. This is clearly started by the authors in Buland and Omre (2003). In Figure 3 a Gaussian probability plot of the compiled logtransformed elastic property data set is presented. The empirical curves do deviate from linearity, indicating non Gaussianity in the underlying distribution. It is particularly so for S-wave velocity.



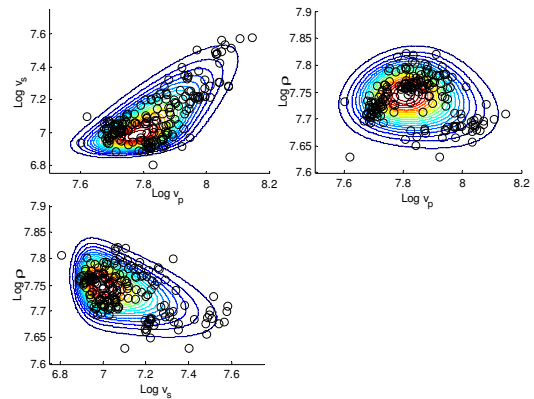
**Figure 3:** Gaussian probability plot of the compiled elastic property data set.

The new contribution of this study is to replace the prior Gaussian model for logtransformed elastic properties by a prior CS-Gaussian model. The latter model captures skewness in the distributions of the elastic properties and also allows analytical solution of the inversion.

### Inversion Methodology

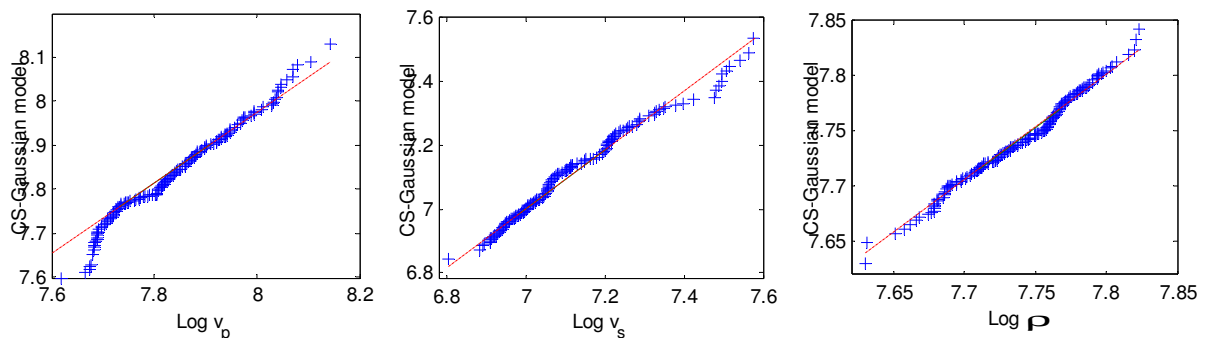
We introduce a Bayesian inversion approach termed Bayesian CS-Gaussian inversion. The theoretical developments can be found in Karimi et al. (2009) and they draw on results concerning closed skew normal distribution, see Gonzalez-Farias et al. (2004). This inversion approach can represent skewness in the distributions of the variables involved. In Bayesian CS-Gaussian inversion, the prior model  $p(\mathbf{m})$  must be CS-Gaussian and the likelihood model  $p(\mathbf{d} | \mathbf{m})$  must be CS-Gauss-linear. The latter entails that  $\mathbf{d}$  and  $\mathbf{m}$  are linearly related with a CS-Gaussian additive error term. If so, the posterior model  $p(\mathbf{m} | \mathbf{d})$  will be CS-Gaussian with model parameters analytically obtainable from the model parameters in the prior and likelihood models. Moreover, all posterior marginal models  $p(\mathbf{m}_i | \mathbf{d})$ ,  $i = 1, \dots, 90$  will be CS-Gaussian with analytically tractable model parameters. The mathematical expressions are complex and can be found Karimi et al. (2009). Note, however, that the expression for the posterior model involves some high-dimensional integrals of Gaussian pdf's which need to be solved numerically.

Lastly, note that the Gaussian model is a particular case of the CS-Gaussian model, hence Bayesian CS-Gaussian inversion is a generalization of Bayesian Gaussian inversion, not merely an alternative inversion approach.



**Figure 4:** Tri-variate CS-Gaussian model (-) and Gaussian model (---), Correlation function and compiled elastic property data (o).

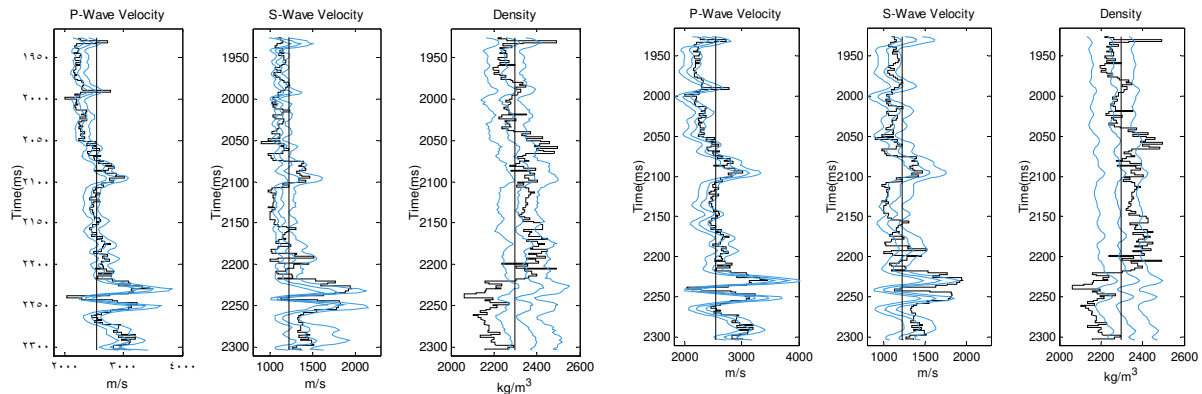
The prior model  $p(\mathbf{m})$  is inferred from the logtransformed elastic property observations in Figure 1, by assuming that all prior marginals  $p(m_i)$ ,  $i = 1, \dots, 90$  are identical. The tri-variate compiled data representing  $\mathbf{m}_i$  is displayed in Figure 4. The model parameters of the tri-variate CS-Gaussian model are estimated by a maximum likelihood approach and the resulting pdf is also displayed in Figure 4. The prior CS-Gaussian model is clearly skewed for S-wave velocity, and deviates clearly from the Gaussian model. A CS-Gaussian probability plot is presented in Figure 5. The empirical curves are more linear than in Figure 3, which is not surprising since the CS-Gaussian is a generalization of the Gaussian. Note, however, that the distribution of P-wave velocity appears with lighter tails than the Gaussian, and that this characteristic is not captured by the generalization. The spatial dependence in the prior model  $p(\mathbf{m})$  is represented by the spatial correlation function in Figure 4. The likelihood model  $p(\mathbf{d} | \mathbf{m})$  is assumed to be Gauss-linear and identical to the model we used for generating the synthetic seismic AVO data.



**Figure 5:** CS-Gaussian probability plot of the compiled elastic property data.

The inversion results from Bayesian CS-Gaussian inversion are presented in Figure 6. For each depth the prediction is defined to be the median of the corresponding posterior marginal pdf. The 0.8-prediction intervals are the [0.1-quantile, 0.9-quantile] of the pdf. Note that these intervals are non-symmetrical around the prediction. Inversion results from traditional Bayesian Gaussian inversion are also presented in Figure 6. Note that median-predictions coincide with expectation-predictions for Gaussian variables. By visual inspection, the Bayesian CS-Gaussian inversion looks better than the traditional inversion approach. In fact, the mean-square-errors for each of the variables for the former

are  $(1.18 \times 10^4, 1.76 \times 10^4, 8.5 \times 10^4)$  while the corresponding values for the latter are  $(2.77 \times 10^4, 2.97 \times 10^4, 9.53 \times 10^4)$ , hence an improvement of almost a factor two.



**Figure 6:** The inversion results from Bayesian CS-Gaussian inversion (left) and Bayesian Gaussian inversion (right). Thick line is median prediction and thin lines are 0.8 prediction interval.

## Conclusion

We have generalized Bayesian Gaussian inversion to Bayesian CS-Gaussian inversion and have implemented it in a case study by comparing the CS-Gaussian model and the Gaussian model, we demonstrate that the CS-Gaussian model is better than the traditional inversion approach. A comparison of the two inversion approaches on real well observations and synthetic seismic AVO data indicates that significant improvements can be obtained.

## Acknowledgement

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