

Elizabeth Stephansen

A pioneer

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Dedicated to the memory of our colleague and friend Alf Aure (1955 – 1994)

It is little known, even in Norway, that the country's first female doctoral candidate was a woman from Bergen by the name of Elizabeth Stephansen. She received her doctoral degree in mathematics at the University of Zürich in 1902. (It was not until 1971 that another Norwegian woman obtained a doctorate in mathematics.) Later Elizabeth Stephansen became an assistant and docent at the Norwegian College of Agriculture at Ås, where she taught until she retired in 1937.

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PART I: Biography

Family and early years

Elizabeth Stephansen's full name was Mary Ann Elisabeth¹ Stephansen. She was born on the 10th of March 1872 in Bergen, the oldest of 7 children of Anton Stephan Stephansen and Gerche Reimers Jahn. Gerche Jahn was the daughter of governmental mediator Otto Georg Jahn and his wife Mary Ann Elizabeth, née Lockwood. The Jahns were of Mecklenburgian ancestry while the Lockwoods were English. Anton Stephansen was the son of a sea captain (For more information on these families see "Røtter & Aner" by Esther Stephansen Nesse). When he was fourteen he was, as he himself said, placed directly from the schoolroom into the well known consul Jebesen's manufacturing business in Bergen. Here he remained eight years until he became a traveling business representative for his employer's factory in Ytre-Arna outside of Bergen. He remained in that position for three years. The young salesman began to dream of owning his own factory, but for that he needed money and money he had little of. As a beginning he decided to start a manufacturing business, and in 1870 he opened his own store on Strandgaten in Bergen and got married soon after. In 1895 the family moved to Espeland, twenty kilometers outside of Bergen, where Stephansen started his own firm A.S. Stephansen A/S or "Janusfabrikken", as the business soon became known after its most popular product.

Betzy, as she was called, therefore grew up in Bergen. On her mother's side she was a Reimers. Representative of Parliament² Otto Reimers was Gerche's cousin. The wife of our national poet Bjørnstjerne Bjørnson, Karoline, née Jahn Reimers, was also Gerche's cousin. The Stephansen and Bjørnson families had good contact with each other. Among other things this friendly contact resulted in many trips to Baden-Baden during the summer. Betzy grew up in a well-to-do family, and it appears that she had a happy childhood free of sorrow. When asked how things were she said "Mary Ann Elizabeth Stephansen, royal subject, always has it good".³

In 1887, five years after the first female student in Norway was

¹She wrote Elisabeth until 1906. Then she began to write her first name with a z as her maternal grandmother had done.

² *Stortingsrepresentant*

³ *Mary Ann Elizabeth Stephansen, kongelig undersått, har det alltid godt*

admitted to the university, three female students started *gymnas*⁴ at Bergen Katedralskole, and one of these was Elizabeth Stephansen. “... I can’t say that we perceived the situation as anything sensational that we were girl students at this boys school”⁵, she wrote in the 1930’s to the school’s principal. “From the school’s perspective there were no extra arrangements made for our sake or much of any furnishings installed for us.”⁶

She attended the science direction at Bergen Katedralskole instead of the classic direction with latin and greek. (This choice was in fact common among women students at the time.) Her sisters and brothers did not attend “gymnas” but had careers from art to engineering. In this family obviously everybody was encouraged to develop his or her talents. The two brothers later joined their father in building up the factory and “Janus-fabrikken” became one of the leading in this branch in Norway.

Out into the world

Elizabeth Stephansen received her *artium*⁷ in 1891, and got her black

⁴Then the Norwegian equivalent of high school (secondary school)

⁵...*Jeg kan ikke si at vi merket det som noe egentlig oppsiktsvekkende at vi var pikeelever ved denne gutteskole*

⁶*Fra skolens side ble det ikke gjort noen som helst ekstra foranstaltninger for vår skyld eller innrettet noe særlig for oss.*

⁷The high school diploma of that time

student hat.

She was a young woman who knew what she wanted. With the support of her family and economic help from her maternal grandfather when her stipend didn't quite stretch far enough, she traveled to Zürich. Her aim was to study at Eidgenössische Polytechnikum (later Eidgenössische Technische Hochschule, ETH) or, more specifically, in the sixth department, "Schule für Fachlehrer in mathematischer und naturwissenschaftlicher Richtung, Mathematische Sektion".

Among the letters left by Karoline Bjørnson is one from Gerche Stephansen dated from the eighth of February, 1892 which provides some insight into what happened: "Betzy, our oldest daughter, as you perhaps know graduated this summer with honors. Her inclination and desire is to study mathematics. If she were to study natural sciences at our university, it would take 6 years and she would have to study many other subjects she has no interest in, since our university is more of an education for government officials than an education in particular subjects. Therefore she was allowed to travel to the polytechnic in Zürich where she was registered in the mathematics department, and did very well on her entry exam, which was very difficult, she was for example the only Norwegian who passed, and over a hundred of

all those who took it failed, of all nationalities of course. Zürich and Paris were the only places where she as a woman could be accepted, we chose Zürich as the polytechnic there is highly recommended and we thought it would be cheaper there than in Paris. But now to the subject at hand. . .”⁸

What the enterprising Gerche wanted was for Bjørnstjerne Bjørnson to put in a good word for Betzy, since she was now applying for money from Queen Josefine’s endowment. Betzy did receive a stipend from the endowment.

The function theorist Adolf Hurwitz was head of ETH’s sixth department. In addition to Hurwitz, Elizabeth Stephansen had, among others, Frobenius as a teacher for differential and integral calculus. She graduated in 1896 with a thesis entitled “Die binäre Form vierten Grades auf dem Kegelschnitte”. Her grades from the courses she followed at Zürich reveal a diligent and capable student. They also show that she had a break in her studies during the school year 1894/95. According to her family this was probably connected with the beginning of Stephansen’s factory.

Home again — temporary teaching positions and doctoral thesis

After she finished her studies in Zürich, Elizabeth Stephansen returned home to Bergen to teach. Even though women had just obtained the right to be employed in secondary schools in 1896 (and from 1912 on the right to take governmental positions, although not clerical positions), it was ten years before the first woman was employed as a tenured teacher in a secondary school. In order to get work women with academic backgrounds applied for teaching jobs which did not require a university education and for temporary teaching positions. See *Alma*

⁸ *Betzy, vor eldste datter, tok som du kanskje vet i sommer realartium med laud. Hendes anlæg og lyst er for matematik. Skulde hun have studeret realfag ved vort universitet, havde det taget 6 aar, og da maatte hun ha lært mange andre fag, som hun ikke vil, da vort universitet jo mere er en uddannelse til embedsmænd, end en uddannelse i fag. Hun fik derfor reise til polyteknikum i Zürich, hvor hun meldtes ind i den matematiske afdeling, og bestod optagelsesprøven meget godt, en meget vanskelig prøve, hun var for eksempel den eneste norske som bestod den, og over hundrede strøg i det hele, af alleslags nationer naturligvis. Zürich og Paris var de eneste steder hvor hun som kvinde kunde blive antaget, vi valgte Zürich da polyteknikummet der er meget anbefalet, og vi troede der var billigere end i Paris. Men nu til sagen.....*

Maters døtre by Stiver Lie and Rørslett. In accordance with this practice Elizabeth Stephansen was first a substitute and part-time teacher at Katedralskolen, and in 1898 she was hired as an assistant at Bergen tekniske skole for a year. Her work consisted of 4 hours of teaching per week for which she was paid 2.50 kroner per hour. (This was incidentally the same pay a man would have received in the same position.) We can get an idea of how the young aspiring engineers reacted to having a female teacher in mathematics and physics—and in second year classes—from a letter written by students in Bergen to their colleagues at Trondhjems tekniske læreanstalt dated the 27th of April 1899: “As a curiosity we can mention that this year they have hired a very attractive young lady as an assistant in second year physics and mathematics since the class this spring was so large it had to be divided into two parallel sessions. To what extent the council of teachers has acted rashly by hiring such a young example of the fair sex, only time will tell. To begin with there are probably many who find that this business of a female teacher for grown technicians, is rather modern, while others so lose their sense when they meet her face to face during examinations that they forget even the simplest rules for good form, not to mention mathematical formulas or what is related thereto. . . ”⁹

In addition to teaching Elizabeth Stephansen worked on a dissertation on partial differential equations, “Über partielle Differentialgleichungen vierter Ordnung die ein intermediäres Integral besitzen”, published in i “Archiv for Mathematik og Naturvidenskab” i 1902. Earlier the same year she had submitted the dissertation for a Doctor of Philosophy degree to the University of Zürich (ETH did not have the authority to give doctorates at that time). Elizabeth Stephansen got her PhD but never had an oral examination. At a meeting in July 1902 Ordinarius Burkhardt, professor in mathematics, suggested that Elizabeth Stephansen be granted her doctorate in absentia. He felt such an action could be defended based on the quality of her work and her

⁹ *Som en kuriositet kan vi berette at der i år er antatt en ung ‘snytvakker’ frøken som assistent i fysikk og matematikk for annen klasse, da denne klasse i vår ble så stor at den måtte deles i to parallellavdelinger. Hvorvidt lærerrådet har handlet lettsindig ved at anta et så ungt individ av det smukke kjønn til lærerinne, får tiden vise. Til en begynnelse er der nok flere som finner at dette med lærerinne for voksne teknikere, er vel moderne, mens andre igjen i den grad taper fatningen når de i eksaminasjonene stilles ansikt til ansikt med frøknen, at de glemmer enda de simpleste regler for god tone, for ikke at tale om de matematiske setninger og hva dermed har forbindelse...*

broad teaching experience. And that is what happened.

To Göttingen!

In 1902 we find an application from Elizabeth Stephansen to the Executive Committee of the Royal Norwegian Frederik's university. The application was dated 27th of February 1902, and she applied for 1000 kroner to study mathematics abroad. Her application was granted with 700 kroner from the State's stipends for scientists' travels abroad¹⁰, and during the winter semester 1902-1903 Stephansen was in Göttingen, where two of the greatest mathematicians at that time, Hilbert and Klein, had their chairs. She followed several lecture series, among them "Mechanik der Continua" taught by Hilbert and "Encyclopädie der Mathematik" taught by Klein, as well as "Versicherungsmathematik" and "Wahrscheinlichkeitsrechnung" taught by *Privatdozent* Zermelo. Inspired by Hilbert's lectures Stephansen wrote an article with the title "Von der Bewegung eines Continuum mit einem Ruhepunkt" published in "Archiv for Mathematik og Naturvidenskap" in 1903.

In a letter from Gerche Stephansen to Karoline Bjørnson we hear that Betzy is often invited to professor Hilbert's home. Hilbert had been to Kristiania (Oslo) during the Abel centenary in 1902, and then visited Karoline's son Bjørn, who clearly had known Hilbert earlier. The Bjørnson family was very much engaged in the cultural activities surrounding Abel's hundredth anniversary. Bjørnstjerne Bjørnson wrote a cantata which is one of the author's most famous poems. In one of the verses he describes mathematics as follows:

| | |
|---------------------------------|---------------------------------|
| <i>Urokkelig som tiden</i> | Unshakable as time |
| <i>er tallenes viden.</i> | is the knowledge of numbers. |
| <i>Deres fletninger er,</i> | Their weavings are, |
| <i>i evig morgenskjær,</i> | in eternal morning light, |
| <i>renere enn sneen,</i> | cleaner than snow, |
| <i>finere enn luften;</i> | finer than air, |
| <i>men sterkere enn verden,</i> | but stronger than the world, |
| <i>som de veier uten skåler</i> | which they weigh without scales |
| <i>og belyser uten stråler.</i> | and illuminate without rays. |

¹⁰ *Statens stipendier for videnskabsmænds reiser i utlandet*

Her life's work at the agricultural college

With her return to Norway Elizabeth Stephansen began to teach again, since she did not receive a further stipend for study abroad. Among those nominated for university stipends in 1903 she was sixth on the list, the mathematician Alf Gulberg was in first place, and the botanist Thekla Resvold in second. In 1904 we find her name in the Oslo address book. The year after we also find her listed as a teacher at Olaf Bergs Pigeskole (Olaf Berg's school for girls) on St. Olav's street in Oslo.

Besides teaching she continued her mathematical studies and wrote two more scientific papers before she was employed in 1906 as an assistant in physics and mathematics at the Agricultural College of Norway.¹¹ She held that position for 15 years, until the 7th of August, 1921 when she was appointed to the newly created docent position in mathematics at the college. She remained in the docent position until the first of July, 1937 when by resolution of the King, she was discharged from her position in accordance with the retirement age law.

The teaching in mathematics at Ås was combined with physics, and Elizabeth Stephansen also assisted the professor in physics. She was otherwise in charge of all of the mathematics being taught for all of the various disciplines of study at the agricultural college. Many of the students did not have the equivalent of a high school education in the natural sciences. Therefore, during the first year students were taught high school mathematics. During the second year students were taught differential and integral calculus and spherical geometry. Only forestry and farmland reapportionment students had mathematics as a mandatory subject. Other students were offered an optimal half year class.

When Elizabeth Stephansen came to the college, descriptive geometry was a subject given a greater number of hours than later. For the undoubtedly difficult task of getting her students to understand the function of the trace in descriptive geometry, her students gave her the nickname "Trasa", a nickname which she inherited from her predecessor (her male predecessor was called "Trasen", the Trace)... "With her Bergen dialect and manners, and as the only woman among her fellow teaching colleagues, it was reasonable that a nickname was be-

¹¹ *Norges Landbrukshøgskole på Ås*

stowed on her — a nickname which I think she was a little proud of, because it tied her perhaps more closely to the college. And she was vitally interested in the success of the college,”¹² wrote a student from Stephansen’s early days at Ås, Olav Klokke. The testimonials from students were very strong. All tell of a pedagogically talented woman who knew her subject. She was well liked and highly respected by her male colleagues as well as by her students. Something which the students remember especially well about “Trasa” was her knitting basket that she always took with her to her lectures, in which she had everything she needed for the day’s work.

As long as Stephansen was at the college she processed the meteorological measurements at Ås and wrote the section *The weather at Ås and underground temperature readings*¹³ in the College’s annual report. Otherwise she was completely consumed with her teaching and there was no time left over for cultivating purely scientific interests. We find her among the participants at the second Scandinavian mathematical congress in Copenhagen in 1911 (for the first congress in Stockholm in 1907 there is no list of participants), but never later. In her application for the docent position she writes that also after her stay in Göttingen she has “taken shorter study trips abroad, primarily in the cause of teaching.”¹⁴

Private Life

Elizabeth Stephansen never married, but according to her niece, she was engaged all of three times! The last time was in 1910 and apparently with a student at Ås. She lived all of her years at Ås in the school’s dormitory “Tivoli” and ate her meals in the building’s dining room along with the students. She was well enough off, but was none the less very thrifty, some would perhaps even say stingy. Jens Aure was one of the students from the last year she taught. He mentioned in a letter that when she now and then treated herself to coffee at the Student union, she wrapped the cubes of sugar which she didn’t use in a paper napkin and took them home with her. Of course that was during

¹² “Med sitt bergenske språk og lynne og som eneste dame i lærerkollegiet, var det ellers rimelig at hun ble påskjønnnet med et utnavn—et utnavn som jeg tror hun var litt stolt av, for det knyttet henne kanskje nærmere til høgskolen. Og høgskolens framgang var hun levende interessert i.” *Morgenposten* 28.2.1961.

¹³ *Været på Ås og temperaturmålingene i jorden*

¹⁴ *Foretatt kortere studiereiser til utlandet, vesentlig i undervisningsøyemed.*

the the “hard thirties”¹⁵, when people of all classes were encouraged to save wherever they could, but to take sugar cubes home with you even then was considered a bit excessive. But that was accepted with a smile when it came to “Trasa”.

Betzy was very fond of her family. She was present at all family celebrations and spent just about all of her summer vacations at Espeland. She enjoyed hiking and never got lost, not even in the thickest fog. When she retired with her pension, she moved to her sister Gerda’s place at Espeland where she lived the rest of her life as the family’s dear and helpful aunt Betzy. During the second world war, with her ability to speak German fluently, she did much for the Norwegian prisoners in the German prison camp at Espeland without consideration for the risks that she took. After the war she received the King’s medal of service for her efforts during the war. In her letter of thanks after her 85th birthday she wrote about her help to the prisoners that it was only common human consideration she gave them, perhaps with a little risk, but all went well. “I think it is nice to have the medal as evidence of a good patriotic attitude”,¹⁶ she writes.

Elizabeth Stephansen died on the 23rd of February, 1961, almost 89 years old. We can find evidence that she was not forgotten by her colleagues or students in the obituaries in *Aftenposten* and *Morgenposten* some days later.

PART II: Scientific work

Mathematical research

Mary Ann Elisabeth Stephansen published four mathematical research papers. Three of them were related to Alf Guldberg’s work on partial differential equations and difference equations. In passing, we mention that Alf Guldberg belonged to a family with many well-known scientists in Norway. He wrote about one hundred research papers in mathematics and statistics. Stephansen’s fourth paper was, as we have heard, inspired by David Hilbert himself.

Über Partielle Differentialgleichungen vierter Ordnung die ein intermediäres Integral besitzen was printed in Christiania in 1902 and

¹⁵ *De harde 30-årene*

¹⁶ *Jeg synes det er godt å ha medaljen som et bevis for god nasjonal holdning*

“begutachtet von Herrn Prof. Dr. Burkhardt” as Inaugural-Dissertation in Zürich. Here Stephansen succeeded in describing all those partial differential equations of the fourth order, in two independent variables, that can be integrated once, i.e., reduced to an equation of the third order. For equations of the second order the reduction had been accomplished by Euler, d’Alembert, and Lagrange. The problem of reducing an equation of the third order to one of the second order had been studied by Alf Guldberg in 1900.

An introduction to problems of this kind is given in Chapter XVI and Chapter XXII in volume VI of Forsyth’s classical treatise. Let us try to describe the mathematical content of Stephansen’s work about equations of the fourth order.

The first observation, obtained by differentiating an arbitrary equation of the third order, was that the fourth order equation necessarily must look like

$$Aa + Bb + Cc + Dd + Ee + F(b^2 - ac) + G(c^2 - bd) \\ + H(d^2 - ce) + I(ad - bc) + K(ae - bd) + L(be - cd) + M = 0$$

in order to have an intermediate integral. If $z = z(x, y)$ is the unknown function, then the lower case letters above stand for the highest derivatives

$$a = \frac{\partial^4 z}{\partial x^4}, b = \frac{\partial^4 z}{\partial x^3 \partial y}, c = \frac{\partial^4 z}{\partial x^2 \partial y^2}, d = \frac{\partial^4 z}{\partial x \partial y^3}, e = \frac{\partial^4 z}{\partial y^4}$$

and A, B, \dots, M are functions of x, y, z and of the first, second, and third partial derivatives of z . These, as well as other abbreviations, were certainly indispensable for the long and elaborate calculations to come. First, Stephansen obtained the necessary condition $IL - FH + GK = 0$.

Second, she arrived at a difficult system of 10 auxiliary equations. These were further reduced to 6 independent auxiliary equations. In a systematic manner five main cases were then considered: “erster Fall”, “zweiter Fall” consisting of five subcases, “dritter Fall” consisting of six subcases, “vierter Fall” consisting of two subcases, and “fünfter Fall”. It is difficult to imagine how even the most degenerate and unfortunate combination of the quantities involved could escape Stephansen’s sophisticated classification scheme: it will be trapped into some subcase.

The special case

$$Aa + Bb + Cc + Dd + Ee + M = 0$$

was analyzed in detail, since considerable simplifications are possible. (Today, this equation is classified as a quasi-linear equation of the fourth order.) Finally, some special cases, where a total integration is possible, were discussed.

The work ends with an explicit example. In order to convey a feeling to the reader for what is going on here we mention this example: The equation

$$\begin{aligned} (1 + s - q)a + (1 + s + p - r)b + (1 + s - r)c \\ - (q + r)d + pe + (b^2 - ac) - (d^2 - ce) \\ - (ad - bc) + (ae - bd) + (be - cd) + p + ps - qr = 0 \end{aligned}$$

has the intermediate integral

$$z + \frac{\partial^3 z}{\partial x^3} + \frac{\partial^3 z}{\partial x^2 \partial y} + \frac{\partial^3 z}{\partial z \partial y^2} = f \left(y + \frac{\partial z}{\partial x} + \frac{\partial^3 z}{\partial x^3} + \frac{\partial^3 z}{\partial y^3} \right),$$

where f is an arbitrary function. (The new letters represent derivatives of lower order.)

It is difficult to judge the scientific merits of Stephansen's contribution to the equations of the fourth order, but it definitely reveals a mathematical talent. Stephansen's acute sense for all kinds of hidden symmetries, not easily detected without clever manipulations with complicated quantities and expressions, was unusual, to say the least. It was perhaps a pity that she did not involve herself in the study of transformation groups. Their theory was developed by Sophus Lie and codified in his book *Theorie der Transformationsgruppen* (1888). Abroad, the subject was very much in the air in the nineties and was to become the main stream for problems of this kind. The celebrated theory of Sophus Lie was still young at that time. As a student of Lie, Alf Guldberg certainly knew about Lie's fundamental discoveries. Today he is remembered in connection with the Vessiot-Guldberg-Lie algebra for integration of nonlinear equations. However, Stephansen did not utilize any of these new methods in her work on equations of fourth order.

The possible mathematical connection between Lie's general theory and Stephansen's piece of work deserves, indeed, to be investigated, but we have not pursued this matter any further.

Stephansen's second paper was *Von der Bewegung eines Continuum mit einem Ruhepunkte* (1903). Though this study has the rather impressive beginning "Aus der von Herrn Prof. Hilbert im Wintersemester 1902-1903 gehaltenen Vorlesung über die Mechanik der Continua habe Ich die Anregung zu der vorliegenden Arbeit genommen", it is merely about a first order linear system of three differential equations with constant coefficients. The determinant of the system is of size 3×3 and it has three eigenvalues, real or complex. The origin is the only point of equilibrium. The solutions are classified by Stephansen in six main cases according to the possible constellations of the three eigenvalues. As always, Stephansen is very systematic, and so case III is divided into four subcases; three of the cases are each divided into two subcases. Needless to say, all possibilities, even the most degenerate, are exhausted here.

The work has a modern flavour. Scientifically, it seems to be the least original of Stephansen's achievements, but this impression is heavily counterbalanced by the present importance of the topic. Today examples of this kind are included in most relevant text-books on differential equations. Indeed, engineering students today would find no difficulties in reading Stephansen's text and they would certainly be able to interpret and appreciate her carefully drawn pictures of the trajectories. — In the unlikely event that Stephansen's work was the first of its kind in three dimensional space, this would be an impressive merit. After a quick search in *Jahrbuch* we cannot make any definite conclusion about priority.

Stephansen published two works about difference equations, one in *Prace Matematyczno-Fizyczne* 1905 and the other one in *Archiv for Matematik og Naturvidenskab* 1906. "Prace" was a mathematical journal printed and edited in Warsaw (Warszawa). It contained original mathematical research as well as translations into Polish of articles of general interest, often written by well-known mathematicians. For example, volume XVI (the one with Stephansen's article) contains a translation of 75 pages of work by G. Mittag-Leffler. Some other illustrious names in "Prace" are K. Weierstrass, T. Levi-Civita, W. Sierpinski, and F. Gomes Teixeira. The contrast between the translations and

the original contributions in this Polish journal was often rather striking. How did it happen that Stephansen found her way to this foreign journal?

The natural explanation seems to be that volume XV (1904) contained a paper by Alf Guldberg (*Über simultane lineare Differenzengleichungen*). Stephansen complements Guldberg's result, taking into account also the degenerate cases. Guldberg had observed that the general solution of the difference system

$$y_{x+1}^{(i)} = A_{i1}y_x^{(1)} + A_{i2}y_x^{(2)} + \cdots + A_{in}y_x^{(n)} \quad (i = 1, 2, \dots, n)$$

with constant coefficients A_{ik} is given by

$$y_x^{(i)} = c_1 e_1^{(i)} a_1^x + c_2 e_2^{(i)} a_2^x + \cdots + c_n e_n^{(i)} a_n^x.$$

Here the c 's are arbitrary constants and a_k is the k^{th} eigenvalue of the matrix (A_{ij}) and $e_k^{(1)}, \dots, e_k^{(n)}$ are the components of the corresponding eigenvector. Symbolically, $Ae_k = a_k e_k$. (The reader does well in regarding the index x as taking the values $1, 2, 3, \dots$) Guldberg's construction yields the general solution only if all the eigenvalues a_1, a_2, \dots, a_n are different. Stephansen's contribution was to treat the degenerate cases. Her observation was that $e_k^{(j)}$ is a rational function of a_k . If, for example, $a_1 = a_2$ then a limiting procedure in Guldberg's formula lead her to the solution

$$y_x^{(i)} = c_1 e_1^{(i)} a_1^x + c_2 \frac{\partial}{\partial a_1} (e_1^{(i)} a_1^x) + c_3 e_3^{(i)} a_3^x + \cdots + c_n e_n^{(i)} a_n^x.$$

Stephansen gave a formula valid in all cases.

It seems appropriate to mention that Alf Guldberg had written several papers on difference equations. Some chapters in Georg Wallenberg's book *Theorie der Linearen Differenzrechnungen* (1911) were written "unter Mitwirkung von Alf Guldberg in Kristiania".

In *Über die symmetrischen Funktionen bei den linearen homogenen Differenzgleichungen* (1906) Stephansen considered a general linear difference equation

$$y_{x+m} + p_1(x)y_{x+m-1} + \cdots + p_m(x)y_x = 0$$

of the m^{th} order. In general, it has m fundamental solutions. (The situation is very similar to that of a linear differential equation of the m^{th}

order.) The main result in Stephansen's work is that any symmetric rational function of the fundamental solutions is a rational function in the coefficients $p_1(x), \dots, p_m(x)$. In other words, symmetric functions of the fundamental solutions can be expressed in terms of known quantities alone. Typical symmetric functions of $y^{(1)}, y^{(2)}, \dots, y^{(m)}$ are for example

$$y^{(1)} + y^{(2)} + \dots + y^{(m)}, \quad \sum_{i \neq j} y^{(i)} y^{(j)}, \quad y^{(1)} y^{(2)} \dots y^{(m)}.$$

Stephansen mentions "Zwei kleine Anwendungen". The second application was the construction of the resultant for two difference equations. She gave the concrete example

$$\begin{cases} y_{x+2} + p_x y_{x+1} + q_x y_x & = 0, \\ y_{x+2} + P_x y_{x+1} + Q_x y_x & = 0 \end{cases}$$

and came to the conclusion that the equations have a solution in common, when

$$Q_x Q_{x+1} + q_x q_{x+1} - Q_x q_{x+1} - q_x Q_{x+1} + p_x p_{x+1} Q_x + P_x P_{x+1} q_x - p_x P_{x+1} Q_x - P_x p_{x+1} q_x = 0.$$

This example was later included in Wallenberg's book. For algebraic equations of the second and third degree similar formulas had been obtained already by Sir Isaac Newton. For comparison we mention that the quadratic equations

$$X^2 + pX + q = 0, \quad X^2 + PX + Q = 0$$

have a common root if and only if

$$Q^2 + q^2 - 2Qq + Qp^2 + qP^2 - PpQ - Ppq = 0,$$

a result that the reader can easily verify, using some high-school algebra.

Reports on weather and temperature

At Ås meteorological observations had been systematically made since 1874 so that a lot of data had been accumulated by 1910. The reports

about the weather at Ås were written by Stephansen for each year in the period 1910–1936 and are published in the annual reports of Norges Landbrukshøiskole. The well-written reports contain a large amount of data, and it must have been a considerable labour to compile the statistics.

In “Beretning om Norges Landbrukshøiskoles virksomhet i budgjetaaret fra 1ste juli 1912 til 30th juni 1913” Stephansen published an interesting study about the mean temperature in the soil beneath the surface of the earth. She used the *annual* variations of the temperature to deduce that the average *daily* variation of the temperature at the depth 0.25m was 1.8°C. At this depth the temperature of the soil was measured daily at 2 o’clock a.m. Obviously, the result depended on the time of the day, and in order to figure out how much, Stephansen calculated the annual mean temperatures over the available period of 16 years at the depths of 0.25m, 0.50m, 1.00m, and 1.50m. The difference of the logarithms of the temperatures at equidistant depths should be constant; in this case it was about 0.21 ± 0.04 . Stephansen used their average 0.2046. From this value she calculated that, at the depth of 15.79m from the surface of the earth, the annual variation of the temperature was 0.01°C, that is, negligible. From this she concluded that the *daily* temperature variation is negligible at the depth of

$$\frac{15.79}{\sqrt{365}}m \approx 0.82m.$$

Then she obtained the result that, at the depth of 0.25m the daily variation of temperature is 1.8°C on the average. Hence the time of the day when the measurements are performed is very decisive at such a low depth¹⁷ — A practical disadvantage, pointed out by Stephansen, was that the corresponding measurements at Strand, a place near Ås, were incommensurable, since they took place at 8 o’clock a.m.

As always, Stephansen gave no references to the literature, but the

¹⁷“Denne værdi forekommer høi, men da man hos os savner alle observationer over temperaturens daglige gang i jorden, maa man indtil videre la den staa for det, den er”.

calculations indicate that she was aware of the content of the formula¹⁸

$$T(y, t) = Ae^{-\sqrt{\pi/KD} \cdot y} \cos\left(\frac{2\pi t}{D} - \sqrt{\frac{\pi}{KD}} \cdot y\right) \\ + Be^{-\sqrt{\pi/KY} \cdot y} \cos\left(\frac{2\pi i}{Y} - \sqrt{\frac{\pi}{KY}} \cdot y\right),$$

essentially due to J. Fourier (1768–1830), who published the influential work “La théorie analytique de la chaleur” 1822.

Let us also mention that the last three annual reports¹⁹ contain a statistical analysis of the weather over a period of 60 years. Stephansen calculated a lot of correlation coefficients. She observed that it is more likely that a cold January one year would be followed by a mild winter next year than not. The correlation is very weak. She found no significant statistical evidence for the common belief that a very cold winter would be followed by an exceptionally warm summer. She also detected some very weak trends in the monthly temperature variations. As always, Stephansen was led by her curiosity and scientific attitude.

Conclusion

Stephansen must be judged as a classical mathematician, continuing a tradition from an earlier epoch (Euler, d’Alembert, Lagrange). Her skilful calculations show a real talent, and a strong scientific attitude is revealed by her continuous interest in the meteorological observations at Ås. Patience and an acute sense for symmetry and order were among her strong qualities. Her style of writing was stringent and elegant.

However, her mathematics is rather isolated and she does not seem to have been aware of any actual trends in the mathematics of her time. Of all her work only the one about the equilibrium point of a linear system of differential equations seems to be of actual interest nowadays. We have been able to find six citations of her work. These are all in the book about difference equations by Alf Guldberg and Wallenberg: Guldberg mentions her three times, so does Wallenberg. Moreover, *Jahrbuch über die Fortschritte der Mathematik* contains reviews 1902, 1904 (probably both written by Alf Guldberg), and 1906 (the author “Dn.” is unknown to us). There is, in addition, a mention in

¹⁸Here $Y = 365D$, $t = \text{time}$, $y = \text{depth}$.

¹⁹1933–34, 1934–35, 1935–36

Jahrbuch 1906. This is, so far, all that we have found. It is not easy to tell how well-known her work was in Norway. Certainly Alf Guldberg had read all her mathematical papers, but, for instance, the item of "Prace" with Stephansen's article that we found in the library of the Department of Mathematics at the University of Oslo was not cut open when we first saw it: no one had read it for 90 years!

PART III: References and acknowledgements

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In conclusion, preparing this report has been a truly interesting and rewarding experience.

Papers written by E. Stephansen²⁰

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