MA2104 Differensiallikninger og kompleks funksjonsteori. Formelark til eksamen

Du må selv vite under hvilke forutsetninger formlene gjelder.

De Moivres formel: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

Cauchy-Riemann likningene: $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}, \quad \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$

Komplekse funksjoner

$$\begin{aligned}
e^z &= \exp z = e^x(\cos y + i\sin y)\\
\log z &= \ln|z| + i\arg z\\
\operatorname{Log} z &= \ln|z| + i\operatorname{Arg} z\\
\cos z &= \frac{1}{2}(e^{iz} + e^{-iz}) = \cos x\cosh y - i\sin x\sinh y\\
\sin z &= \frac{1}{2i}(e^{iz} - e^{-iz}) = \sin x\cosh y + i\cos x\sinh y
\end{aligned}$$

Cauchys integralformel: $f(z_0) = \dfrac{1}{2\pi i}\displaystyle\int_C \frac{f(z)}{z - z_0}\,dz$

Cauchys generelle integralformel: $f^{(n)}(z_0) = \dfrac{n!}{2\pi i}\displaystyle\int_C \frac{f(z)}{(z - z_0)^{n+1}}\,dz$

Noen potensrekker:

$$\begin{aligned}
\frac{1}{1-z} &= 1 + z + z^2 + z^3\cdots\\
e^z &= 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!}\cdots\\
\cos z &= 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!}\cdots\\
\sin z &= z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!}\cdots
\end{aligned}$$

Jordans Lemma:

$$\int_0^\pi e^{-R\sin\theta}\,d\theta \le \frac{\pi}{R}$$

Fourierrekker

Periodisk funksjon $f(x)$ med periode $2L$:

$$
\begin{aligned}
f(x) \sim & a_0 + \sum_{n=1}^{\infty} (a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L}) \\
a_0 = & \frac{1}{2L} \int_{-L}^{L} f(x)\, dx \\
a_n = & \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L}\, dx \\
b_n = & \frac{1}{L} \int_{-L}^{L} f(x) \sin \frac{n\pi x}{L}\, dx
\end{aligned}
$$

Sinus- og cosinus-rekker for funksjonen $f(x)$ gitt i intervallet $(0, L)$:

$$
\begin{aligned}
f(x) \sim & \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}, \\
b_n = & \frac{2}{L} \int_{0}^{L} f(x) \sin \frac{n\pi x}{L}\, dx \\
f(x) \sim & a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}, \\
a_0 = & \frac{1}{L} \int_{0}^{L} f(x)\, dx \quad a_n = \frac{2}{L} \int_{0}^{L} f(x) \cos \frac{n\pi x}{L}\, dx
\end{aligned}
$$

Noen integral:

$$
\begin{aligned}
&\int x e^x\, dx = (x-1)e^x + C \\
&\int x^n e^x\, dx = x^n e^x - n \int x^{n-1} e^x\, dx + C \\
&\int \ln x\, dx = x \ln x - x + C \\
&\int x^n \ln x\, dx = \frac{x^{n+1} \ln x}{n+1} - \frac{x^{n+1}}{(n+1)^2} + C \\
&\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2+b^2} (a \sin bx - b \cos bx) + C \\
&\int e^{ax} \cos bx\, dx = \frac{e^{ax}}{a^2+b^2} (a \cos bx + b \sin bx) + C \\
&\int \sin^2 x\, dx = \frac{x}{2} - \frac{\sin 2x}{4} + C \\
&\int \cos^2 x\, dx = \frac{x}{2} + \frac{\sin 2x}{4} + C \\
&\int x \sin x\, dx = -x \cos x + \sin x + C \\
&\int x \cos x\, dx = x \sin x + \cos x + C \\
&\int x^n \sin x\, dx = -x^n \cos x + n \int x^{n-1} \cos x\, dx \\
&\int x^n \cos x\, dx = x^n \sin x - n \int x^{n-1} \sin x\, dx
\end{aligned}
$$