4.6.4

$$f(z) = \sin \frac{1}{z}$$

$$f(z) = 0 \Rightarrow \sin \frac{1}{z} = 0 \Rightarrow \frac{1}{z} = k\pi \Rightarrow z = \frac{1}{k\pi} \quad k \neq 0$$

$$f'(z) = \cos \frac{1}{z} \cdot \left(-\frac{1}{z^2}\right)$$

$$f'\left(\frac{1}{k\pi}\right) = -k^2\pi^2 \cos k\pi \neq 0$$

$\Rightarrow$ $z = \frac{1}{k\pi}$ for $k \neq 0$ er isolerte nullpunkt av orden 1.

7.4.15

Sinus-rekkeutviklinga til $f(x) = e^x$ for $0 < x < 1$ er

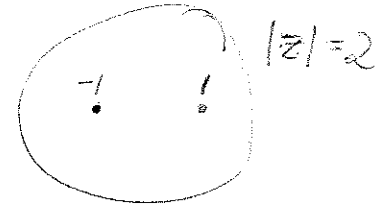
$$\sum_{n=1}^{\infty} b_n \sin n\pi x$$

der

$$\begin{aligned}
b_n &= 2\int_0^1 f(x) \sin(n\pi x)\, dx \\
&= 2\int_0^1 e^x \sin(n\pi x)\, dx \\
&= 2\left[\frac{e^x}{1+n^2\pi^2}\left(\sin(n\pi x) - n\pi\cos(n\pi x)\right)\right]_0^1 \\
&= 2\left(\frac{e}{1+n^2\pi^2}\left(-n\pi\cos(n\pi)\right) - \frac{1}{1+n^2\pi^2}\left(-n\pi\cos 0\right)\right) \\
&= \frac{2n\pi}{1+n^2\pi^2}\left(1-(-1)^n e\right)
\end{aligned}$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{2n\pi}{1+n^2\pi^2}\left(1-(-1)^n e\right)\sin(n\pi x)$$

Eksamen mai 2000 oppg 3



a) $\frac{z^2+1}{z^2-1} = -\frac{1}{z+1} + 1 + \frac{1}{z-1}$

$\int_{C_2(0)} \frac{dz}{z+1} = \int_{C_r(-1)} \frac{dz}{z+1} = 2\pi i$ Cauchy's integralformel

$\int_{C_2(0)} 1 \cdot dz = 0$ siden 1 er analytisk

$\int_{C_2(0)} \frac{1}{z-1}\,dz = \int_{C_r(1)} \frac{dz}{z-1} = 2\pi i$ Cauchy's integralformel

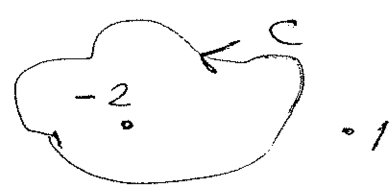
Så $\int_{C_2(0)} \frac{z^2+1}{z^2-1}\,dz = -2\pi i + 0 + 2\pi i = 0$

b) Integrasjonsveien blir

Så $\int_{\gamma} \frac{dz}{z-1} = \int_{C_r(1)} \frac{dz}{z-1} = 2\pi i$

de to andre integralene blir 0 siden 1 og $\frac{1}{z+1}$ er analytiske i det indre av γ.

c) Nevner $= 0$ for $z = 1$ og $z = -2$



$\int_C = \int_{C_r(-2)}$ Sett $f(z) = \frac{z^2}{1-z}$, Da er f analytisk i det indre av C og

$I = \int_C \frac{z^2\,dz}{(1-z)(z+2)^2} = \int_C \frac{f(z)}{(z+2)^2}\,dz = \frac{f'(-2)}{2\pi i}$ ved Cauchy's gen. integralformel

$f'(z) = \frac{2z - z^2}{(1-z)^2}$ $f'(-2) = -\frac{8}{9}$ så $I = \frac{-16\pi i}{9}$