8.3.5

$$\star \begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} & 0 < x < 1 \\ u(0,t) = u(1,t) = 0 & \forall\, t > 0 \\ u(x,0) = x & 0 < x < 1 \end{cases}$$

Ved seperasjon av variable finn ein at $\star$ har løysinga

$$u(x,t) = \sum_{n=1}^{\infty} b_n e^{-n^2\pi^2 t} \sin n\pi x$$

der

$$b_n = 2\int_0^1 x \sin n\pi x \, dx$$

$$= 2\left[\frac{1}{n^2\pi^2} \sin n\pi x - \frac{x}{n\pi} \cos n\pi x \right]_0^1$$

$$= \frac{-2}{n\pi} \cos n\pi = \frac{2 \cdot (-1)^{n+1}}{n\pi}.$$

$$\Rightarrow \quad u(x,t) = \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n\pi} e^{-n^2\pi^2 t} \sin n\pi x$$