Øving 4

Oppg 4.5.18

$$f(z) = \frac{4z-5}{(z-2)(z-1)}$$

$$\frac{4z-5}{z-1} = 4 - \frac{1}{z-1}$$

$$\frac{1}{z-1} = \frac{1}{1+(z-2)} = \frac{1}{z-2}\,\frac{1}{\left(1+\frac{1}{z-2}\right)}$$

$$= \frac{1}{z-2}\sum_{n=0}^{\infty}\left(\frac{-1}{z-2}\right)^n = \sum_{n=0}^{\infty}\frac{(-1)^n}{(z-2)^{n+1}}$$

der $\quad |z-2|>1$

$$f(z) = \frac{4}{z-2} - \frac{1}{z-2}\sum_{n=0}^{\infty}\frac{(-1)^n}{(z-2)^{n+1}}$$

$$= \frac{4}{z-2} + \sum_{n=2}^{\infty}\frac{(-1)^{n-1}}{(z-2)^n} \qquad |z-2|>1$$

4.5.25

$$\sin\frac{1}{z} = \frac{1}{z} - \frac{1}{3!}\frac{1}{z^3} + - \dots$$

$$\int_{C_1(0)} \sin\frac{1}{z}\,dz = \int_{C_1(0)} \frac{1}{z}\,dz - \frac{1}{3!}\int_{C_1(0)} \frac{1}{z^3}\,dz + \dots$$

$$= \underline{2\pi i} - 0 + 0 - \dots$$

Oppg 7.1.8

$$f(x) = \cos x + \cos \pi x$$

$$f(x) = 2 \iff \cos x = 1 \text{ og } \cos \pi x = 1$$

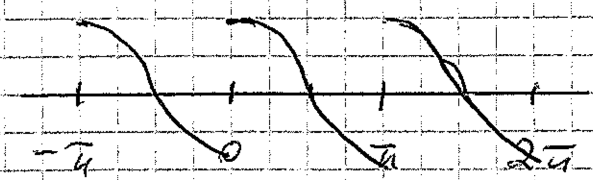
$$\iff x = 2m\pi \text{ og } \pi x = 2n\pi$$

$$\iff x = 2n = 2m\pi \implies \pi = \frac{n}{m}$$

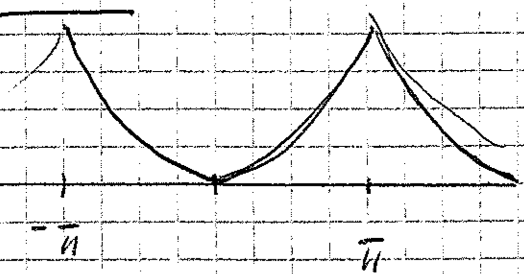
Siden π er irrasjonal, må $n = 0$ der $x = 0$.

7.1.12

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\,dx = \int_{0}^{\pi} f(x)\,dx = 0$$



7.2.9



f er kont, stykkevis glatt så Fourierrekke konv mot $f(x)$ for alle x.

Eks MA214 1994

$$f(z) = \frac{e^z}{z^2(1+z^2)} \quad \text{se på } g(z) = \frac{e^z}{1+z^2}$$

a) $e^z = 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \dots$

$$\frac{1}{1+z^2} = 1 - z^2 + z^4 - \dots$$

$$\frac{e^z}{1+z^2} = \left(1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \dots\right)\left(1 - z^2 + z^4 - \dots\right)$$

(produkt av rekker, ikke pensum nå)

$$= 1 + z + \frac{z^2}{2} + \frac{z^3}{6} - z^2 - z^3 + \text{ledd av orden 4 og større}$$

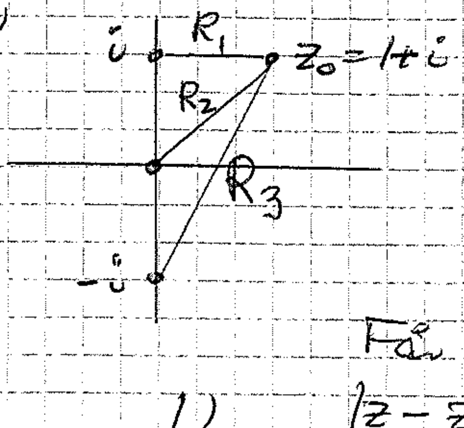
dvs $= 1 + z - \frac{1}{2}z^2 - \frac{5}{6}z^3 + \dots$

Så

$$f(z) = \frac{g(z)}{z^2} = \frac{1}{z^2} + \frac{1}{z} - \frac{1}{2} - \frac{5}{6}z + \dots$$

$\pm i$ blir nærmeste singulære punkt, så Laurentrekka gjelder for $0 < |z| < 1$.

b)



Singulære pkt er

$z_1 = i \quad z_2 = 0 \quad z_3 = -i$

avstand fra $z_0 = 1+i$ blir

$R_1 = 1 \quad R_2 = \sqrt{2} \quad R_3 = \sqrt{5}$

Får da 4 forskjellige Laurentrekker

1) $|z - z_0| < 1$

2) $1 < |z - z_0| < \sqrt{2}$

3) $\sqrt{2} < |z - z_0| < \sqrt{5}$

4) $\sqrt{5} < |z - z_0|$