Bayesian Lithology/Fluid Inversion Constrained by Rock Physics Depth Trends and a Markov Random Field

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Abstract

In this study rock physics depth trends and a lithology/fluid Markov random field are used to constrain a 2D Bayesian lithology/fluid inversion. Lithology/fluid classes are defined in a lithology/porosity/fluid space, and a stochastic relation from lithology/porosity/fluids to prestack seismic data is established. The rock physics depth trends are modeled by porosity depth trends, which also can be considered as low frequency prior information in the seismic inversion. The posterior model is the objective and block Gibbs samplers are used to make inference about the posterior model. The inversion algorithm is tested on a real data set from the North Sea, and the classification results are encouraging. The posterior model is also used to predict seismic elastic properties, and the results are compared to predictions from a simpler method. Due to the discrete lithology/fluid classes the estimate of the seismic elastic parameters are piecewise smooth and high contrasts are identifiable.

Introduction

Elastic properties of rocks are strongly influenced by geological trends, and by estimating these trends the uncertainties in the lithology/fluids prediction can be reduced. In this study the AVO classification and depth trends in (Avseth et al., 2003) are considered in a spatial setting with a Markov random field (MRF) as prior model for the lithology/fluids. The full study is presented in (Rimstad and Omre, 2009).

Model

Consider a cross section of a sedimentary layered reservoir, where the main objective is to classify lithology/fluids denoted by $\boldsymbol{\pi}$ which is defined in a lithology/porosity/fluid space. To connect lithology/porosity/fluids and seismic prestack data a stochastic model is defined. The model is a combination of (Avseth et al., 2003), (Buland and Omre, 2003) and (Ulvmoen, 2007). The lithologies considered are mixtures of clay and quartz, and the fluids are gas and brine. Let $\boldsymbol{\pi}_{tx} \in \{\mathrm{SandGas}, \mathrm{SandBrine}, \mathrm{Shale1}, \mathrm{Shale2}\}$. Shale1 and Shale2 have different clay volume.

Estimation of the global likelihood parameters $\boldsymbol{\tau}$ is also an objective. The global likelihood parameters are parameters that do not vary spatially: porosity depth trend $\boldsymbol{\lambda}$, wavelets $\mathbf{w}$ and covariance matrices $\Sigma_{\mathbf{m}}, \Sigma_{\mathbf{d}}$. They are denoted $\boldsymbol{\tau} = [\boldsymbol{\lambda}, \mathbf{w}, \Sigma_{\mathbf{m}}, \Sigma_{\mathbf{d}}]$.

The observations are $\mathbf{o} = [\mathbf{d}, \mathbf{o}_w]$, where $\mathbf{d}$ are seismic prestack data and $\mathbf{o}_w$ well data. The well data is all the information available for the well trace, both well log and seismic data. The model is graphically displayed in Figure 1.

The inversion is solved in a Bayesian framework; hence the posterior model $p(\boldsymbol{\pi} \mid \mathbf{o})$ is the objective and can generally be written as

$$\begin{aligned} p(\boldsymbol{\pi} \mid \mathbf{o}) &= \int p(\boldsymbol{\pi}, \boldsymbol{\tau} \mid \mathbf{o})\, \mathrm{d}\boldsymbol{\tau} \\ &\approx \int p(\boldsymbol{\pi} \mid \mathbf{d}, \boldsymbol{\tau})\, p(\boldsymbol{\tau} \mid \mathbf{o}_w)\, \mathrm{d}\boldsymbol{\tau}, \qquad (1) \end{aligned}$$

$$p(\boldsymbol{\pi} \mid \mathbf{d}, \boldsymbol{\tau}) = \mathrm{const} \times \int p(\mathbf{d} \mid \mathbf{m}, \boldsymbol{\tau})\, p(\mathbf{m} \mid \boldsymbol{\pi}, \boldsymbol{\tau})\, \mathrm{d}\mathbf{m}\, p(\boldsymbol{\pi}). \qquad (2)$$

where $\mathbf{m}$ is the logarithm of the elastic properties: p-wave, s-wave and density. Then $[\boldsymbol{\pi} \mid \mathbf{o}]$ in Expression 1 can be sampled approximately by sequential simulation, first sample $\boldsymbol{\tau}$ from $p(\boldsymbol{\tau} \mid \mathbf{o}_w)$ and then sample $\boldsymbol{\pi}$ from $p(\boldsymbol{\pi} \mid \mathbf{d}, \boldsymbol{\tau})$. The prior models for $\boldsymbol{\pi}$ and $\boldsymbol{\tau}$ are assumed to be independent. In Expression 2, $p(\mathbf{d} \mid \mathbf{m}, \boldsymbol{\tau})$ is the seismic likelihood, $p(\mathbf{m} \mid \boldsymbol{\pi}, \boldsymbol{\tau})$ is the rock physics likelihood and $p(\boldsymbol{\pi})$ is a prior model for the lithology/fluids.

In the lithology/porosity/fluid space the classes are depth dependent, and the depth trends in the model are porosity depth trends. Both compaction and cementation trends are included. The porosity $\phi(t)$ is the expected value of the porosity at depth t and it is decomposed and parameterized as in (Ramm and Bjørlykke, 1994):

$$\phi_{sh}(t) = \phi^0_{sh} \exp\left\{-\alpha_{sh}(t - t^0)\right\}, \qquad (3)$$

$$\phi_{ss}(t) = \begin{cases} \phi^0_{ss} \exp\left\{-\alpha_{ss}(t - t^0)\right\} & \text{if } t \leq t^c_{ss} \\ \phi_{ss}(t^c_{ss}) - \kappa_{ss}(t - t^c_{ss}) & \text{if } t > t^c_{ss} \end{cases}, \qquad (4)$$

where sh indicates shale, ss indicates sand, t^0 is a reference depth, $\phi^0_{\cdot}$ is the porosity at depth t^0, the cementation initiates at depth t^c_{ss}, and $\alpha_\bullet$ and κ_{ss} are regression coefficients. The depth to the top of the reservoir is denoted t^0. The porosity depth trends are illustrated in Figure 2. It is assumed that there is a known linear relation between the depth in time and meter. Let c_1 be the clay content in Shale1 and c_2 be the clay content in Shale2, and

$$\boldsymbol{\lambda} = \left[\phi^0_{sh}, \phi^0_{ss}, \alpha_{sh}, \alpha_{ss}, \kappa_{ss}, t^c_{ss}, c_1, c_2\right]. \qquad (5)$$

The rock physics likelihood $p(\mathbf{m} \mid \boldsymbol{\pi}, \boldsymbol{\tau})$ is constructed from a Hashin-Shtrikman Hertz-Mindlin model for unconsolidated sand (Avseth et al., 2005), a Hashin-Shtrikman shale model for shale (Holt

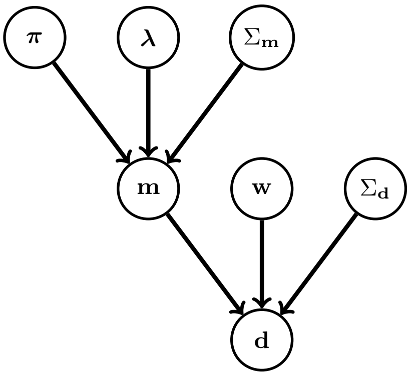

Figure 1: *Stochastic model. The nodes represent stochastic variables and the arrows represent probability dependencies.*

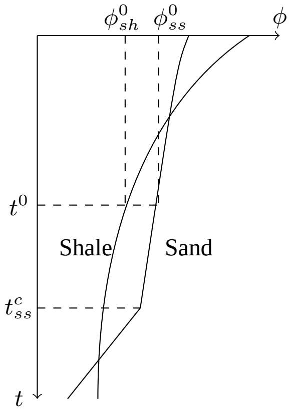

Figure 2: *Porosity, ϕ, with respect to depth, t, for shale and sand.*

and Fjær, 2003) and Dvorkin-Nur constant/contact cement model for cemented sandstone (Dvorkin and Nur, 1996). Fluid effects are calculated by Gassmann's relations (Gassmann, 1951).

The seismic likelihood $p(\mathbf{d} \mid \mathbf{m}, \boldsymbol{\tau})$ is based on a linearized weak contrast approximation of the Zoeppritz equation and is on the form

$$[\mathbf{d} \mid \mathbf{m}, \boldsymbol{\tau}] = WAD\mathbf{m} + \mathbf{e_d}, \qquad \mathbf{e_d} \sim N(0, \Sigma_{\mathbf{d}}), \tag{6}$$

where W is a convolution matrix based on the wavelets, A is a reflection matrix and D is a differential matrix (Buland and Omre, 2003).

In the prior model for lithology/fluids $p(\boldsymbol{\pi})$ is defined as a profile MRF (Ulvmoen, 2007). Hence the lithology/fluid in the position tx given all the other lithology/fluids $\boldsymbol{\pi}_{-tx}$ is only dependent on the neighbors:

$$p(\pi_{tx} \mid \boldsymbol{\pi}_{-tx}) = p(\pi_{tx} \mid \pi_{(t-1)x}, \pi_{(t+1)x}, \pi_{t(x-1)}, \pi_{t(x+1)}), \tag{7}$$

where $\boldsymbol{\pi}_{-tx}$ is $\boldsymbol{\pi}$ without π_{tx}. The MRF provide a natural ordering of lithology/fluids, for example that brine never is directly above gas.

The model for the global parameters given the well observations $p(\boldsymbol{\tau} \mid \mathbf{o})$ is designed such that each parameters can conditionally be sampled by Gibbs samplers. The posterior model is obtained by sampling $p(\boldsymbol{\tau} \mid \mathbf{o})$ and $p(\boldsymbol{\pi} \mid \mathbf{d}, \boldsymbol{\tau})$ with a MCMC algorithm.

Example

Consider a real North Sea data set. The well log is illustrated in Figure 3 and the prestack seismic data is available at three angles, one of them is illustrated in Figure 4. The well log is at trace 101. In this model it is assumed that there are no cementation trends, hence t_{ss}^c is larger than the depth of the reservoir.

The classification results are presented in Figure 4 and the MAP-prediction look very trustworthy. Note that the brine have a fairly natural location below the gas. The probability maps represent the uncertainty in the classifications. On the left and right boundary there appear to be some boundary problems. The posterior model for the porosity depth trends parameters $\boldsymbol{\lambda}$ is plotted in Figure 5. The prior models for the porosity depth trends parameters are uniform and invisible in the figure, and the posterior variance are significantly reduced compared to the prior model.

In Figure 6 estimates of the seismic elastic parameters in trace 145 are plotted. Two different models are used. One is based on (Buland and Omre, 2003), Model A, and the other is based on the model presented in this study where the discrete lithology/fluid classes are included, Model B. The norm of

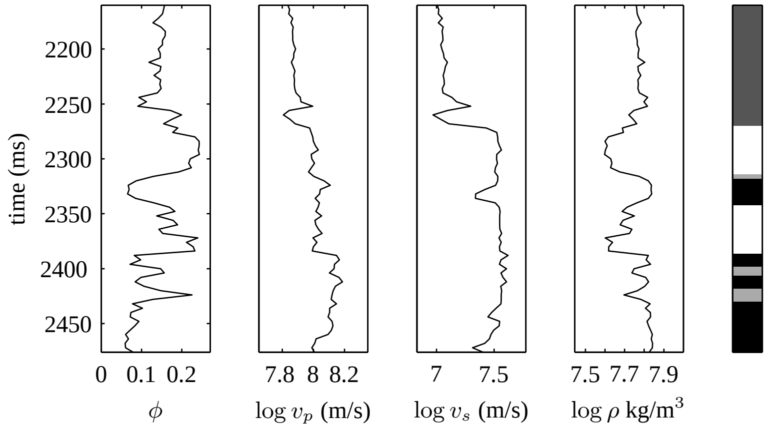

Figure 3: *Well log. From left: porosity, seismic elastic properties and lithology/fluids. White is SandGas, light gray is SandBrine, dark gray is Shale1 and black is Shale2.*

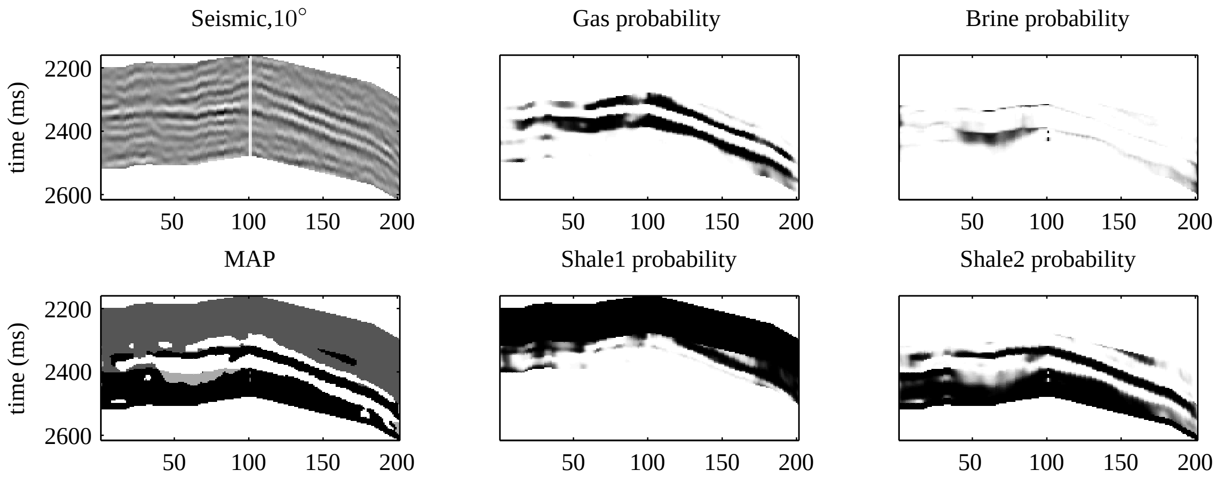

Figure 4: *Seismic observations and classification results, well: trace 101. MAP: maximum a posteriori estimate, and SandGas, SandBrine, Shale1, Shale2 probability plots, black is high probability and white is low.*

the variance in Model B is reduced with about 65% compared to Model A. The use of the discrete lithology/fluid classes and the parameterized porosity trends acts as a filter for the prediction of the seismic elastic parameters and cause them to have a stepwise structure and high contrasts. It is crucial that the rock physics models are as reliable as possible in the inversion, both for the classification and the prediction of seismic elastic parameters. To get better fit more parameters could be included, but then overfitting could be a problem.

Conclusions

In this study a stochastic relationship from lithology/porosity/fluids to prestack seismic data is established. The seismic lithology/fluid inversion is solved in a Bayesian framework constrained by rock physics models and a prior profile MRF. The results from the study are encouraging. It is also shown that the posterior model can be used to predict seismic elastic parameters. The methods are demonstrated on a reservoir in two dimensions, but the methodology may easily be extended to three dimensions.

Acknowledgements

The research is part of the Uncertainty in Reservoir Evaluation (URE) activity at the Norwegian University of Science and Technology (NTNU). Discussions with P. Avseth and R. Holt were important for the study.

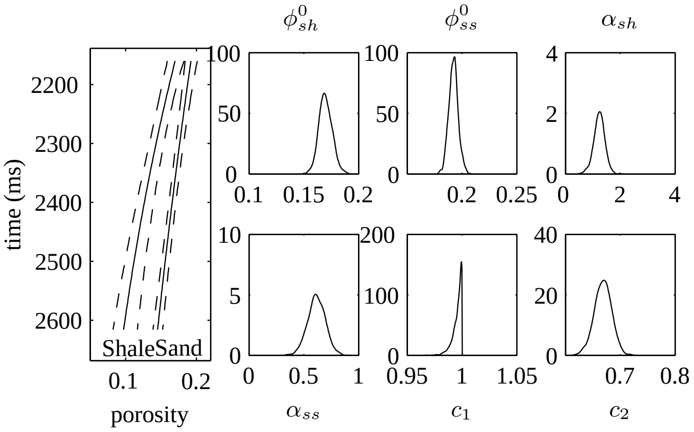

Figure 5: *Depth trend parameters. Left: posterior models for $\phi_{sh}(z), \phi_{ss}(z)$, black is posterior mean and dashed black is 95% posterior confidence interval. Right: posterior models for the $\boldsymbol{\lambda}$-components.*

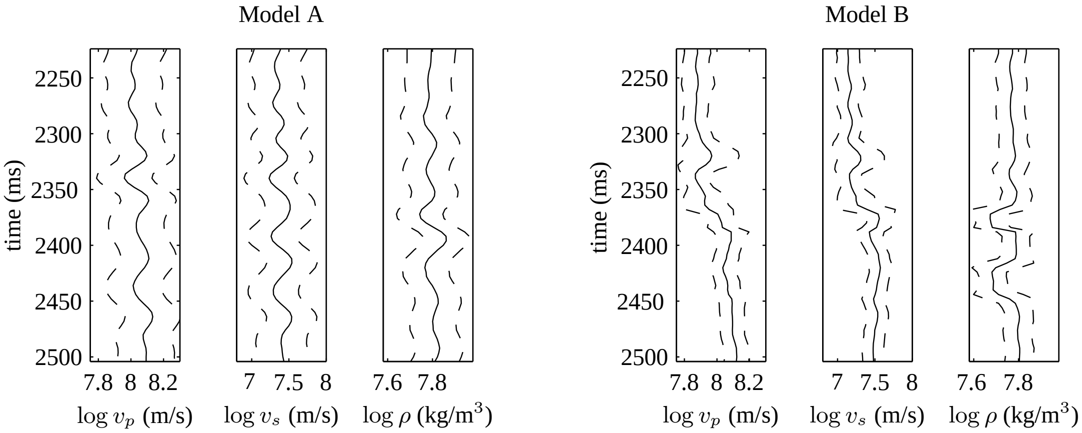

Figure 6: *Comparing Method A (left) and Method B (right). Solid black is the prediction and the dashed black is a 0.95 prediction interval.*

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