MA3201 - Problem sheet 3

Time and place: Wednesday 6th October $14^{\underline{15}} - 16^{\underline{00}}$ in F3.

If there is some time left after having done the problems, we continue with the lectures.

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Problem 1. Let F be a field, and let

$$R = \left\{ \begin{pmatrix} a & 0 & 0 \\ b & \alpha & 0 \\ c & 0 & \alpha \end{pmatrix} \mid a, b, c, \alpha \in F \right\}.$$

(a) Show that R is a ring with 1. Is R commutative?

(b) Let $J = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ b & 0 & 0 \\ c & 0 & 0 \end{pmatrix} \mid b, c \in F \right\}$. Show that J is a two-sided ideal in R, and that $R/J \simeq F \times F$.

(c) Let $e = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. Show that Re is a maximal left ideal in R, og find all left ideals in R which are contained in Re.

Challenge. Let Λ_c be the algebra over $\mathbb{C}$ with basis $\{e_0, e_1, e_2, e_3\}$, where c is a given complex number. The multiplication is given by the following rule:

	e_0	e_1	e_2	e_3
e_0	e_0	e_1	e_2	e_3
e_1	e_1	e_3	e_3	0
e_2	e_2	$-e_3$	ce_3	0
e_3	e_3	0	0	0

Find for which c and c' the algebras Λ_c and $\Lambda_{c'}$ are isomorphic.