QPA EXERCISE SHEET, MA3203

PROBLEM 1

(a) Construct the quiver Q and the path algebra $A = kQ$, where k is the rationals and Q is the quiver

$$Q: \qquad 1 \xrightarrow{a} 2 \xrightarrow{b} 3$$

The generators of A are stored as `A.v1`, `A.v3`, `A.v3` (the three lazy paths corresponding to vertices 1, 2 and 3), `A.a` and `A.b` (the arrows). You can make linear combinations of the generators by writing for example `t := 2*A.v2 + A.a`.

(b) Construct the following (right) A-module:

$$M: \qquad k^2 \xrightarrow{\binom{1}{1}} k \xrightarrow{0} 0$$

(c) Try the following commands to get information about and compute with M and its elements.

- `DimensionVector(M)` gives the dimensions of the vector spaces in the representation (that is, if the dimension vector of some A-module is $(1, 0, 2)$, then the module is on the form $k \to 0 \to k^2$).
- `Basis(M)` gives a basis for M. `Basis(M)[i]` gives the i^{th} basis element. You can make any element of M using the basis, for example `m := Basis(M)[1] + 2*Basis(M)[2]`.
- Given some element p of the algebra and m of the module, we compute mp by using the hat symbol: `m^p`. Do this for some elements.
- You can examine the matrices of M by writing

  `MatricesOfPathAlgebraMatModule(M),`

  but note that you can not directly use the output to construct a new module if the zero vector space occurs at some position in M.

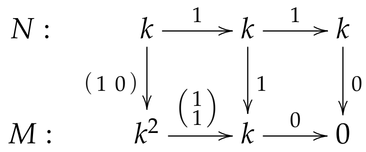
PROBLEM 2

We look at the same quiver Q, algebra A and module M as in the previous problem, in addition to the module N given by

$$N: \qquad k \xrightarrow{1} k \xrightarrow{1} k$$

Date: March 2013.

(a) Construct $f : N \to M$ where f is given by



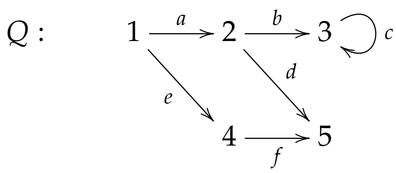
(b) Find the kernel and cokernel of f. Use `DimensionVector` and `MatricesOfPathAlgebraMatModule` to verify that you have got the correct modules.

(c) Using the method `HomOverPathAlgebra(`*`Mod1`*`,`*`Mod2`*`)`, find a basis for $\mathrm{Hom}_A(N, M)$. The method call returns a list of the basis elements (you can access them with the `[]` operator).

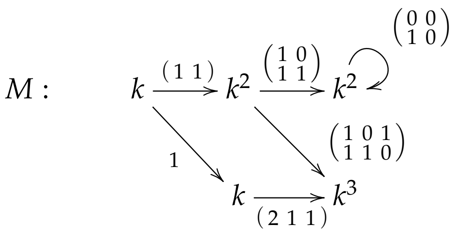
(d) Find a basis for $\mathrm{End}_A(M, M)$ and $\mathrm{End}_A(N, N)$. Use this to decide whether the modules are decomposable or not.

PROBLEM 3

Given the quiver



and the relations $\rho = \{c^2, ad - ef\}$, find the radical of the $kQ/\langle\rho\rangle$-module M where



The radical is computed by the command `RadicalOfModule(M)`. Calculating by hand, it is not that hard to spot the dimension vector of the radical, but the maps are harder to find.

PROBLEM 4

Use `ProjectiveCover(M)` to find the projective cover of M. The function returns the map from the projective to M. Use `Source(`*`map`*`)` to find the projective itself. Find the top of M by using `TopOfModule(M)`.

The indecomposable projectives can be found using the command

`IndecomposableProjectiveModules(`*`algebra`*`)`.

They are given in the same order as the vertices.

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(PROBLEM 5)

Use QPA to do the following exercises on Problem sheet 4: 1 and 2.