MA3203 - Problem sheet 3

Problem 1. Given $\Lambda = k\Gamma/\langle\rho\rangle$, where Γ is the quiver $1 \xrightarrow{\alpha} 2 \underset{\gamma}{\overset{\beta}{\rightrightarrows}} 3$ with relations $\rho = \{\beta\alpha\}$ and k is a field. Find the radicals and tops of representations of Λe_i for different possible values of i.

Problem 2. Given $\Lambda = k\Gamma/\langle\rho\rangle$, where Γ is the quiver $1 \underset{\beta}{\overset{\alpha}{\rightleftarrows}} 2$ with relations $\rho = \{\alpha\beta\}$ and k is a field. Let J be the ideal in $k\Gamma$ generated by the arrows.

(a) Show that there is some t such that $J^t \subset \langle\rho\rangle \subset J^2$. What is the dimension of Λ over k?

(b) Find the representations of Λe_i for different possible values of i and find their radicals and tops.

(c) Find the radical of Λ.

Problem 3. Let $\mathfrak{r}$ be the radical of a ring Λ, and let

$$A \xrightarrow{f} B \xrightarrow{g} C \to 0$$

be an exact sequence of Λ-modules and homomorphisms (i.e. $\operatorname{Im} f = \operatorname{Ker} g$ and g is onto; f does not need to be mono).

Show that the sequence

$$A/\mathfrak{r}A \xrightarrow{\bar{f}} B/\mathfrak{r}B \xrightarrow{\bar{g}} C/\mathfrak{r}C \to 0$$

also is exact, where the maps $\bar{f}$ and $\bar{g}$ are induced by $\bar{f}(a + \mathfrak{r}A) = f(a) + \mathfrak{r}B$ and $\bar{g}(b) = g(b) + \mathfrak{r}C$.