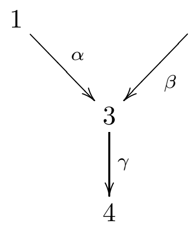
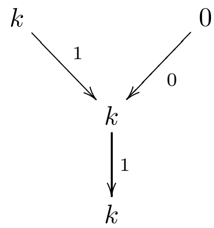
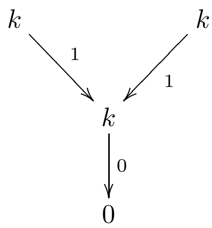
MA3203 - Midterm test

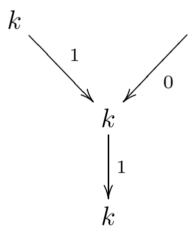
The midterm test consists of 3 problems.

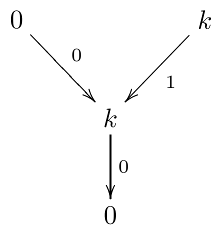
Problem 1. Given $\Lambda = k\Gamma/\langle\rho\rangle$, where Γ is the quiver 

with relations $\rho = \{\gamma\beta\}$ and k is a field.

(a) Find the representations corresponding to Λe_1, Λe_2, Λe_3 and Λe_4.
(b) Find a composition series of the Λ-module Λe_1. What is the length of Λe_1?
(c) Find all the homomorphisms from the representation 

to the representation 

, and from the representation 

and to the representation 

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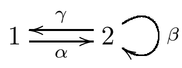
Problem 2. Given $\Lambda = k\Gamma/\langle\rho\rangle$, where Γ is the quiver 

with relations $\rho = \{\beta\alpha, \gamma\beta, \beta^2 - \alpha\gamma\}$ and k is a field.

(a) Find the dimension of Λ as a vector space over k.
(b) Let J be the ideal in $k\Gamma$ generated by the arrows. Show that $J^t \subset \langle\rho\rangle \subset J^2$ for some $t \geq 2$.
(c) For the left Λ-modules Λe_1 and Λe_2 find the corresponding representations of (Γ, ρ). Find the radical of the representations.
(d) Find the radical $\mathfrak{r}$ of Λ. What is the smallest positive integer n such that $\mathfrak{r}^n = (0)$. Let $M = \mathrm{rad}(\Lambda e_2)$. Find the top of M.

Problem 3. Let Λ be a finite dimensional algebra over a field k. Let M be a left Λ-module. Show that the following are equivalent.

(a) M is a finitely generated Λ-module.
(b) M has finite length.
(c) M as a vector space over k is finite dimensional.