

MA3203 - PROBLEM SHEET 1

Problem 1.

- (a) Given a quiver $\Gamma: 1 \longrightarrow 2 \longrightarrow 3$ and a field k , let $\Lambda = k\Gamma$. Let S be the k -algebra $\begin{pmatrix} k & 0 & 0 \\ k & k & 0 \\ k & k & k \end{pmatrix}$. Show that Λ and S are isomorphic as k -algebras.
- (b) Given the representation $(V, f): k^2 \xrightarrow{(1 \ 1)} k \xrightarrow{\begin{pmatrix} 0 \\ 1 \end{pmatrix}} k^2$ and the representation $(V', f'): k \xrightarrow{\begin{pmatrix} 1 \\ 0 \end{pmatrix}} k^2 \xrightarrow{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}} k^2$ of Γ over k , describe homomorphisms from (V, f) to (V', f') .
- (c) Describe homomorphisms from (V', f') to (V, f) where (V, f) and (V', f') are as in (b).
- (d) We say that a representation (V, f) of some quiver Γ over some field k is a subrepresentation of a representation (V', f') of Γ over k if there is a monomorphism from (V, f) to (V', f') .

Let (V', f') be the representation $k \xrightarrow{1} k \xrightarrow{1} k$ of the quiver Γ from (a) over the field k . Find all subrepresentations of (V', f') .

- (e) We say that a representation (V, f) of some quiver Γ over some field k is a factor of a representation (V', f') of Γ over k if there is an epimorphism from (V', f') to (V, f) .

Let (V', f') be as in (d). Find all factor-representations of (V', f') .

Problem 2. Let $R = \begin{pmatrix} k & 0 \\ k[x]/(x^2) & k \end{pmatrix} \subset \begin{pmatrix} k[x]/(x^2) & 0 \\ k[x]/(x^2) & k[x]/(x^2) \end{pmatrix}$, where k is a field and addition and multiplication are the usual addition and multiplication of matrices.

- (a) Show that R is a ring, and that it is an algebra over k when $\alpha \cdot r = \begin{pmatrix} \alpha & 0 \\ 0 & \alpha \end{pmatrix} r$ for $\alpha \in k$ and $r \in R$. Is R a left artinian ring?
- (b) Let $I = \begin{pmatrix} 0 & 0 \\ k[x]/(x^2) & 0 \end{pmatrix}$. Show that I is an ideal in R . Is R a semisimple ring? Show that $R/I \cong k \oplus k$ as rings.
- (c) Show that R is isomorphic to the path algebra of the quiver $\Gamma: 1 \xrightarrow[\beta]{\alpha} 2$.
- (d) Let M_λ be the representation of the quiver Γ over k given by $V(1) = k, V(2) = k, f_\alpha = 1$ and $f_\beta = \lambda$ (i.e. $f_\alpha(a) = a, f_\beta(a) = \lambda a, \forall a \in k$) where $\lambda \in k$. Show that $M_\lambda \not\cong M_{\lambda'}$ if $\lambda \neq \lambda'$.
- (e) Show that M_λ is indecomposable for each λ in k .

Problem 3.

- (a) Given the quiver $\Gamma: 1 \overset{\circlearrowleft}{\curvearrowright} \alpha$ with a relation $\sigma = \alpha^2$ and a field k , let $\Lambda = k\Gamma/\langle \alpha^2 \rangle$. Show that Λ and $k[x]/(x^2)$ are isomorphic as k -algebras.
- (b) Find all homomorphisms from the representation $(V, f): k^3 \overset{\circlearrowleft}{\curvearrowright} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ to itself.
- (c) Find all indecomposable representations of (Γ, σ) over k . Hint: Only two!