Deloooping Moravian Maps
Stable and Unstable Operations in the Morava K–theories
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21st British Topology Meeting

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12th September 2006
1. When is an unstable cohomology operation a component of a stable one?
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2. If we have a component of a stable operation, can we construct the other components?
Outline

The Questions

Preliminaries

Reconstructing Stable Operations

K–theory Mod p

Morava K–theories
Preliminaries

Let \( E^*(-) \) be a graded, generalised cohomology theory.
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Contravariant functor $E^*(-) : \text{Top} \to \text{GAb}$

- $X$ topological space $\leadsto E^*(X)$, graded abelian group.
- $f : X \to Y$ continuous $\leadsto f^* : E^*(Y) \to E^*(X)$ of graded abelian groups (degree zero), with $(fg)^* = g^*f^*$. 
Let $E^*(-)\) be a **graded, generalised cohomology theory**.

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- **$X$ topological space** $\leadsto E^*(X)$, **graded abelian group**.
- **$f : X \rightarrow Y$** continuous $\leadsto f^* : E^*(Y) \rightarrow E^*(X)$ of graded abelian groups (degree zero), with $(fg)^* = g^*f^*$.
- **$E^*(-)$ intertwines suspensions**: $E^k(\Sigma X) \cong (\Sigma E^*(X))^k = E^{k-1}(X)$, natural in $X$. 
Forgetfulness

Three views of $E^*(-)$:
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- A family of functors, $\{E^k(-)\}$, into sets.
An operation is a natural transformation between functors.
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\[ F, G : C \to \mathcal{D} \] contravariant.

\( \nu : F \to G \) is:

for every \( C \)–object \( X \), \( \nu_X : F(X) \to G(X) \) such that:

\[
\begin{align*}
F(X) & \xrightarrow{\nu_X} G(X) \\
F(f) \uparrow & \quad \Downarrow \quad \uparrow G(f) \\
F(Y) & \xrightarrow{\nu_Y} G(Y)
\end{align*}
\]
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\[
S^h \to \mathcal{A}^{k+h}_k \subseteq \mathcal{U}^{k+h}_k
\]
Examples

- Coefficient operations on $E^*(-)$: $n(x) = nx$;
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- Multiplication operations on $H^*(-)$: $x \mapsto x^k$
- Steenrod squares on $H^*(-; \mathbb{F}_2)$.
- Bott periodicity in $K$–theory:
  $\beta : K^{k+2}(X) \xrightarrow{\cong} K^k(X)$;
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- Multiplication operations on $H^*(-)$: $x \mapsto x^k$;
- Steenrod squares on $H^*(-; \mathbb{F}_2)$;
- Bott periodicity in $K$–theory:
  \[ \beta : K^{k+2}(X) \xrightarrow{\cong} K^k(X); \]
- Adams operations in $K$–theory: for $k \in \mathbb{Z}$,
  \[ \Psi^k : K^0(X) \rightarrow K^0(X). \]
  \[ \Psi^k(L) = L^\otimes k, \Psi^k(V \oplus W) = \Psi^k(V) \oplus \Psi^k(W). \]
1. When is an unstable cohomology operation a component of a stable one?

2. If we have a component of a stable operation, can we construct the other components?
Consider an unstable operation:

\[ r : E^k(\cdot) \to E^l(\cdot) \]
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\[ r : E^k(-) \to E^l(-) \]

Define a new operation:

\[ \Omega r : E^{k-1}(-) \to E^{l-1}(-) \]

by:

\[ (\Omega r)_X : E^{k-1}(X) \cong E^k(\Sigma X) \xrightarrow{r_{\Sigma X}} E^l(\Sigma X) \cong E^{l-1}(X) \]
Proposition

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Mild help: often have a uniqueness theorem.
Example: K–theory

K–theory:

2–periodic.

\[ K^{-2}(X) \quad K^{-1}(X) \quad K^0(X) \]
\[ \downarrow \quad \downarrow \quad \downarrow \]
\[ K^1(X) \quad K^2(X) \quad K^{-2}(X) \quad K^{-1}(X) \quad K^0(X) \quad K^1(X) \]

Question: \( r = \Omega (\beta - 1)^r \beta \)?
Example: K–theory

K–theory:

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Example: K–theory

K–theory:

\[
\begin{align*}
K^{-2}(X) \quad K^{-1}(X) \quad K^0(X) \quad K^1(X) \quad K^2(X) \\
\downarrow \Omega^2 r \quad \downarrow \Omega r \quad \downarrow r \\
K^{-2}(X) \quad K^{-1}(X) \quad K^0(X) \quad K^1(X) \quad K^2(X)
\end{align*}
\]
Example: K–theory


\[
\begin{array}{cccccc}
K^{-2}(X) & K^{-1}(X) & K^0(X) & K^1(X) & K^2(X) \\
\Omega^2 r & \Omega r & r & & \\
K^{-2}(X) & K^{-1}(X) & K^0(X) & K^1(X) & K^2(X) \\
\end{array}
\]

\[\beta \rightarrow \beta^{-1}\]
Example: K–theory


\[
\begin{array}{cccccc}
K^{-2}(X) & K^{-1}(X) & K^0(X) & K^1(X) & K^2(X) \\
\downarrow \Omega^2 r & \downarrow \Omega r & \downarrow r & \downarrow \beta^{-1} r \beta \\
K^{-2}(X) & K^{-1}(X) & K^0(X) & K^1(X) & K^2(X) \\
\end{array}
\]
Example: $K$–theory


\[
\begin{array}{cccccc}
K^{-2}(X) & K^{-1}(X) & K^0(X) & K^1(X) & K^2(X) \\
\Omega^2 r & \Omega r & r & \Omega(\beta^{-1} r \beta) & \beta^{-1} r \beta \\
K^{-2}(X) & K^{-1}(X) & K^0(X) & K^1(X) & K^2(X)
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Example: K–theory


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K^{-2}(X) & K^{-1}(X) & K^{0}(X) & K^{1}(X) & K^{2}(X) \\
\Omega^2 r & \Omega r & r & \Omega(\beta^{-1}r\beta) & \beta^{-1}r\beta \\
K^{-2}(X) & K^{-1}(X) & K^{0}(X) & K^{1}(X) & K^{2}(X) \\
& & \beta^{-1} & & \\
& & & & \\
& & & & \\
\end{array}
\]

Question: \( r = \Omega^2(\beta^{-1}r\beta) \)?
Adams Operations

$$\Omega^2 (\beta^{-1} \Psi^k \beta) = k \Psi^k$$
Adams Operations

\[ \Omega^2(\beta^{-1}\Psi^k \beta) = k\Psi^k \]

\[
\begin{array}{ccc}
K^{-2}(X) & K^0(X) & K^2(X) \\
\downarrow \Psi^k & & \\
K^{-2}(X) & K^0(X) & K^2(X)
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<table>
<thead>
<tr>
<th>(K^{-2}(X))</th>
<th>(K^0(X))</th>
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<tbody>
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<td>(k\Psi^k)</td>
<td>(\Psi^k)</td>
<td></td>
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</table>
Adams Operations

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\[ \begin{array}{ccc}
K^{-2}(X) & \rightarrow & K^0(X) & \rightarrow & K^2(X) \\
\downarrow k \Psi^k & & \downarrow \Psi^k & & \downarrow \frac{1}{k} \Psi^k \\
K^{-2}(X) & \rightarrow & K^0(X) & \rightarrow & K^2(X)
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Adams Operations

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\downarrow k\Psi^k & \downarrow \Psi^k & \downarrow \frac{1}{k}\Psi^k \\
K^{-2}(X) & K^0(X) & K^2(X)
\end{array}
\]

\[ \frac{1}{k}\Psi^k \text{ not an operation on } K^0(-) \]
(Unless \( k = 1 \) or \( k = -1 \))
How to divide by $k$: introduce coefficients. $R$ a commutative, unital ring $K(-; R)$ K–theory with coefficients in $R$. 
Coefficients

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Examples

1. $R = \mathbb{Q}$
Coefficients

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Examples

1. $R = \mathbb{Q}$, but $K^*(-; \mathbb{Q}) \cong H^\pm(-; \mathbb{Q})$
Coefficients

How to divide by \( k \): introduce coefficients. \( R \) a commutative, unital ring \( K(−; R) \) \( K \)-theory with coefficients in \( R \).

Examples

1. \( R = \mathbb{Q} \), but \( K^*(−; \mathbb{Q}) \cong H^\pm(−; \mathbb{Q}) \)
2. \( R = \mathbb{Z}_{(p)} \) retains \( p \)-typical information
   \( \Psi^k \) is stable if \( p \nmid k \)
   (see work of Clarke, Crossley, and Whitehouse)
Warning: $K(X; \mathbb{F}_p) \neq K(X)/(p)$.
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$p = 3, \ k = 11$

\[\begin{array}{ccc}
K^0(X; \mathbb{F}_p) & K^2(X; \mathbb{F}_p) & K^4(X; \mathbb{F}_p) \\
\downarrow \Psi^{11} & & \\
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\end{array}
$$
$\Psi^{11}$ repeats with period 4
$\Psi^{11}$ repeats with period $4 = 2(3 - 1)$
Answers

\[ \Psi^{11} \text{ repeats with period } 4 = 2(3 - 1) \]

In \( K^*(-; \mathbb{F}_p) \), for \( p \nmid k \), \( \Psi^k \) repeats with period \( 2(p - 1) \) (Fermat)
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(Fermat)

**Proposition**

**In** \( K^*(-; \mathbb{F}_p) \):

1. \( \Psi^k \) is stable if and only if \( p \nmid k \);
$\Psi^{11}$ repeats with period $4 = 2(3 - 1)$

In $K^*(-; \mathbb{F}_p)$, for $p \nmid k$, $\Psi^k$ repeats with period $2(p - 1)$ (Fermat)

**Proposition**

In $K^*(-; \mathbb{F}_p)$:

1. $\Psi^k$ is stable if and only if $p \nmid k$;
2. If $\Psi^k$ is stable the (even) components are blocks of:

   $$(\Psi^k, kp^{-2}\Psi^k, kp^{-3}\Psi^k, kp^{-4}\Psi^k, \ldots, k\Psi^k).$$
Theorem (S – Whitehouse)

The components of a stable operation on $K^*(-; \mathbb{F}_p)$ repeat with periodicity $2(p - 1)$. 
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The components of a stable operation on $K^*(-; \mathbb{F}_p)$ repeat with periodicity $2(p - 1)$.

Corollary

If $\Omega^{2(p-1)}(\beta^{-(p-1)}r\beta^{p-1}) = r$ then $r$ is a component of a stable operation.
Start with a component of a stable operation.

\[ K^k(X; \mathbb{F}_p) \xrightarrow{r_k} K^{k+h}(X; \mathbb{F}_p) \]
Reconstruction

Start with a component of a stable operation.

\[
\begin{align*}
K^k(X; \mathbb{F}_p) & \quad K^l(X; \mathbb{F}_p) \\
\downarrow r_k & \quad \downarrow r_l \\
K^{k+h}(X; \mathbb{F}_p) & \quad K^{l+h}(X; \mathbb{F}_p)
\end{align*}
\]
Reconstruction

Start with a component of a stable operation.

\[
\begin{align*}
K^i(X; \mathbb{F}_p) & \xrightarrow{\beta^{m(p-1)}} K^k(X; \mathbb{F}_p) \xrightarrow{r_k} K^{l}(X; \mathbb{F}_p) \\
K^{i+h}(X; \mathbb{F}_p) & \xleftarrow{r_l} K^{k+h}(X; \mathbb{F}_p) \xleftarrow{\beta^{-m(p-1)}} K^{l+h}(X; \mathbb{F}_p)
\end{align*}
\]
Start with a component of a stable operation.

$K^i(X; \mathbb{F}_p) \xrightarrow{\Omega^j r_k} K^i+h(X; \mathbb{F}_p)$

$K^k(X; \mathbb{F}_p) \xrightarrow{r_k} K^{k+h}(X; \mathbb{F}_p)$

$K^l(X; \mathbb{F}_p) \xrightarrow{r_l} K^{l+h}(X; \mathbb{F}_p)$

$\beta^{m(p-1)} \xrightarrow{\beta^{-m(p-1)}}$
Reconstruction

Start with a component of a stable operation.

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K^{k+h}(X; \mathbb{F}_p) & \xrightarrow{r_l} K^{l+h}(X; \mathbb{F}_p) \\
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Theorem (S – Whitehouse)

Let $r$ be an unstable operation on $K^*(-; \mathbb{F}_p)$. Then $r$ is a component of a stable operation if (and only if) there is an unstable operation $s$ with $r = \Omega s$. 
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Let $r$ be an unstable operation on $K^*(-; \mathbb{F}_p)$. Then $r$ is a component of a stable operation if (and only if) there is an unstable operation $s$ with $r = \Omega s$.

That is, if $r$ deloops once then it deloops as many times as we like.
Morava K–theories

For each prime $p$, a sequence of cohomology theories $\{K(n)^*(-)\}$ – the **chromatic filtration**.
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- \(K(0)^*(-) = H^*(-; \mathbb{Q})\)
Morava $K$–theories

For each prime $p$, a sequence of cohomology theories $\{K(n)^*(-)\}$ – the **chromatic filtration**.

- $K(0)^*(-) = H^*(-; \mathbb{Q})$
- $K(1)^*(-)$ summand of $K^*(-; \mathbb{F}_p)$
For each prime $p$, a sequence of cohomology theories \( \{K(n)^*(-)\} \) – the **chromatic filtration**.

- \( K(0)^*(-) = H^*(-; \mathbb{Q}) \)
- \( K(1)^*(-) \) summand of \( K^*(-; \mathbb{F}_p) \)
- \( K(n)^*(-): \)
  - is periodic, period \( 2(p^n - 1) \)
  - has coefficients \( \mathbb{F}_p[\nu_n, \nu_n^{-1}], |\nu_n| = -2(p^n - 1) \)
  - has Künneth formula and duality
Theorem (S – Whitehouse)

1. The components of a stable operation in $K(n)^*(-)$ repeat with periodicity $2(p^n - 1)$;
Theorem (S – Whitehouse)

1. The components of a stable operation in $K(n)^*(-)$ repeat with periodicity $2(p^n - 1)$;
2. If $r$ is an unstable operation such that there is another unstable operation $s$ with $r = \Omega s$ then $r$ is a component of a (unique) stable operation.
1. The periodicity has changed to reflect the periodicity of the cohomology theory.
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2. The periodicity is always that of the cohomology theory. (Compare with $K^*(-; \mathbb{F}_p)$)
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2. The periodicity is always that of the cohomology theory. (Compare with $K^*(-; \mathbb{F}_p)$)

3. The “delooping” condition has not changed: if we can deloop once we can deloop as many times as we like.
1. Projection $P : \mathcal{U}_k^l \rightarrow \mathcal{U}_k^l$ via:

$$Pr = \Omega^{2(p^n-1)}(v_n^{-1}rv_n)$$

such that $r$ is a component of a stable operation if and only if $r = Pr$. 
Remarks

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3. Reconstruction is easy using periodicity.
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such that $r$ is a component of a stable operation if and only if $r = Pr$.

2. Closely linked to the Bousfield–Kuhn functor.

3. Reconstruction is easy using periodicity.

4. Proof is a straightforward analysis of the $p$–series of the formal group law.
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2. If we have a component of a stable operation, can we construct the other components?
Questions and Answers

For the Morava $K$–theories:

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Questions and Answers

For the Morava K–theories:

1. When is an unstable cohomology operation a component of a stable one?

   If it can be delooped once.

2. If we have a component of a stable operation, can we construct the other components?

   Yes; easily, using the periodicity.
Delooping Moravian Maps

Stacey, Whitehouse

The Questions

Preliminaries

Cohomology Theories

Operations

Reconstructing Stable Operations

Looping

Example: K–theory

Coefficients

Mod p

Answers (Adams Operations)

All Operations

Reconstruction

Recognition

Morava K–theories

Reconstruction and Recognition

Notes

Remarks

Questions (conclusion)

Navigation