Professor Dirac's Cookbook

or

How to Construct a Dirac Operator in Infinite Dimensions

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In Today's Program

Recipe of the Day

Cooking up a Dirac operator

Ingredients Under the Microscope

Orthogonal structures

Grow Your Own

co-Riemannian structure

Recipe of the Day

The Basic Recipe

Ingredients

Remarks

Method

The Basic Recipe

Ingredients

1. Smooth manifold, M

Remarks

oriented, spin, even dimensional, ...

Method

The Basic Recipe

Ingredients

1. Smooth manifold, M
2. Riemannian structure, g

Remarks

$$g_p \quad T_pM \quad T_pM$$
$$g_p \quad T_pM \quad\quad T_pM$$

Method

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1. Add g to M and leave until a connection appears.

The Basic Recipe

Ingredients

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2. Riemannian structure, g
3. Spin structure, Q

Remarks

2 Spin_n SO_n
2 Q P

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2. Place Q over the mixture and allow to infuse upwards.

The Basic Recipe

Ingredients

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4. Spin representations, S

Remarks

Has Clifford Multiplication

$c \quad {}^{n} \quad S^{+} \quad S$ (bilinear)

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Clifford Multiplication becomes

$$c \quad TM \quad S_M^+ \quad S_M$$

(fibrewise bilinear)

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Remarks

$$/ \ (S_M^+) \quad ((TM, S_M^+))$$

$$(TM \ S_M^+)$$

$$g^1 \ (TM \ S_M^+)$$

$$c \ (S_M)$$

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First Variation

Ingredients

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infinite dimensional,
polarised, oriented, spin ...

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Ingredients

1. Smooth manifold, X
2. Riemannian structure, g
3. Spin structure, Q

Remarks

$$\begin{matrix} S^1 & \mathrm{Spin}_J & \mathrm{SO}_J \\ S^1 & Q & P \end{matrix}$$

Method

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First Variation

Ingredients

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Remarks

No longer canonical

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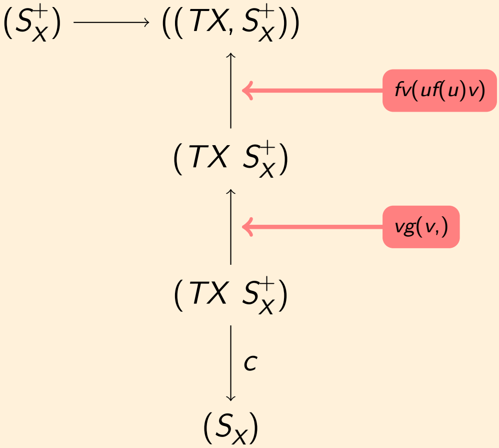
Result

Total collapse of Dirac soufflé.

The Collapse



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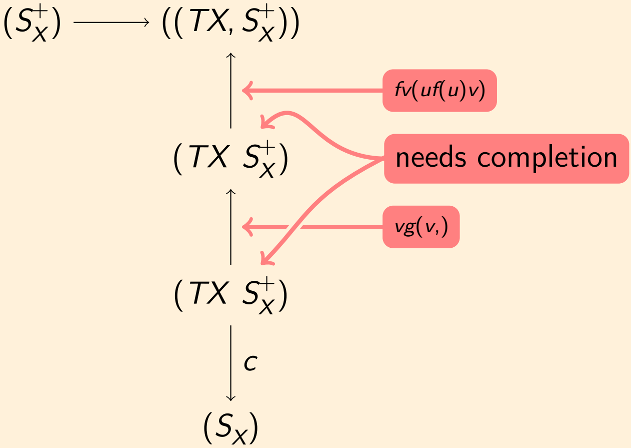

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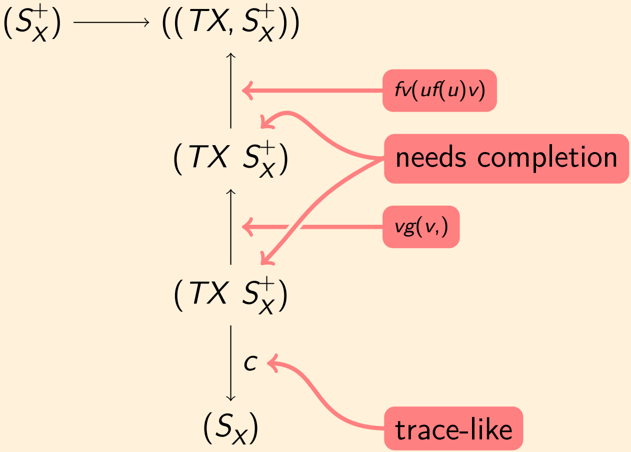

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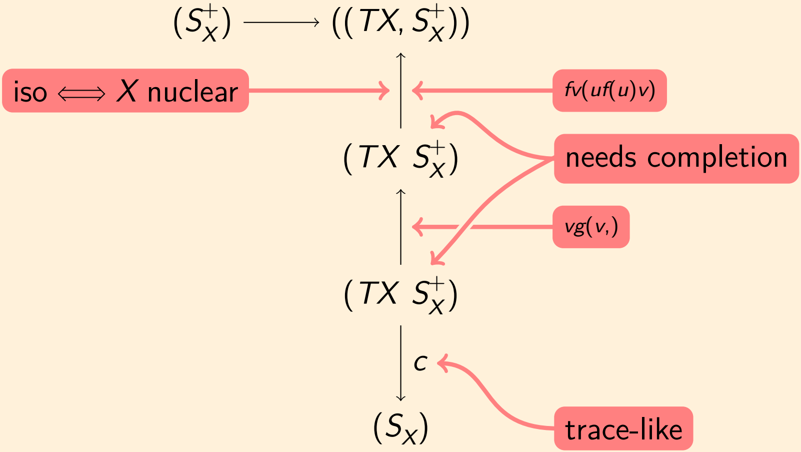

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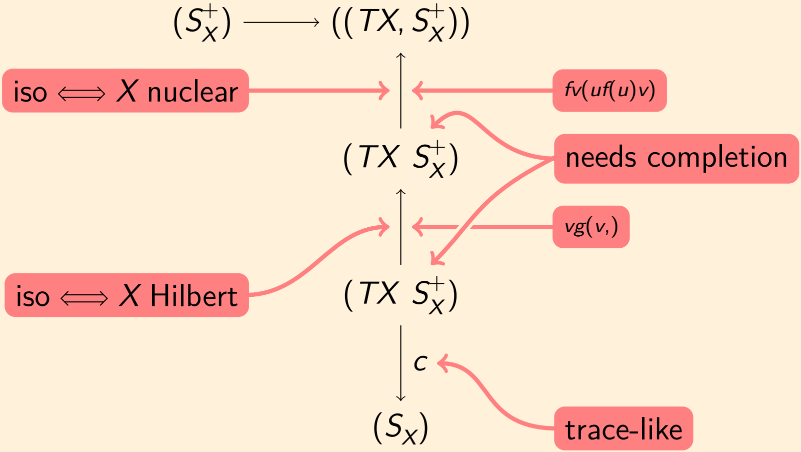

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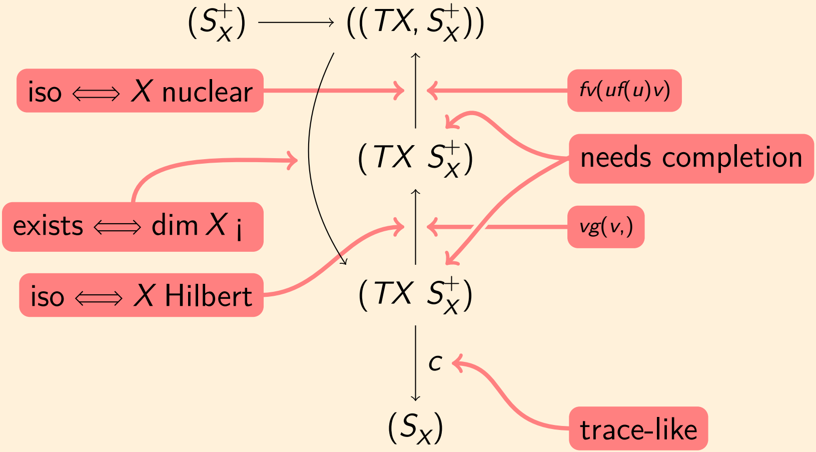

The Improved Basic Recipe

Ingredients

Remarks

Method

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Ingredients

1. Smooth manifold, M

Remarks

oriented, spin, even dimensional, ...

Method

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Ingredients

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$g_p \quad T_pM \quad T_pM$
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1. Add g to M and leave until a connection appears.

The Improved Basic Recipe

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1. Smooth manifold, M
2. Riemannian structure, g
3. Spin structure, Q

Remarks

$$\begin{matrix} _2 & \mathrm{Spin}_n & \mathrm{SO}_n \\ _2 & Q & P \end{matrix}$$

Method

1. Add g to M and leave until a connection appears.

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1. Smooth manifold, M
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4. Spin representations, S

Remarks

Build from ^{n}

Has Clifford Multiplication

c ^{n} S^+ S (bilinear)

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Clifford Multiplication becomes $c\ TM\ S_M^+\ S_M$ (fibrewise bilinear)

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$$/ \ (S_M^+) \quad ((TM, S_M^+))$$

$$(TM \ S_M^+)$$

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Remarks

Isomorphism

$g \quad TM \quad TM$

gives equivalence

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Second Variation

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infinite dimensional,
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Method

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Second Variation

Ingredients

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Remarks

No longer canonical

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$c \quad V \quad S^{+} \quad S$ (bilinear)

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Clifford Multiplication
becomes
$c \; TX \; S_X^+ \; S_X$
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Second Variation

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$$\not\ \ (S_X^+) \quad ((TX, S_X^+))$$
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$$^c\ (S_X)$$

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Question

Does it collapse?

Second Variation

Ingredients

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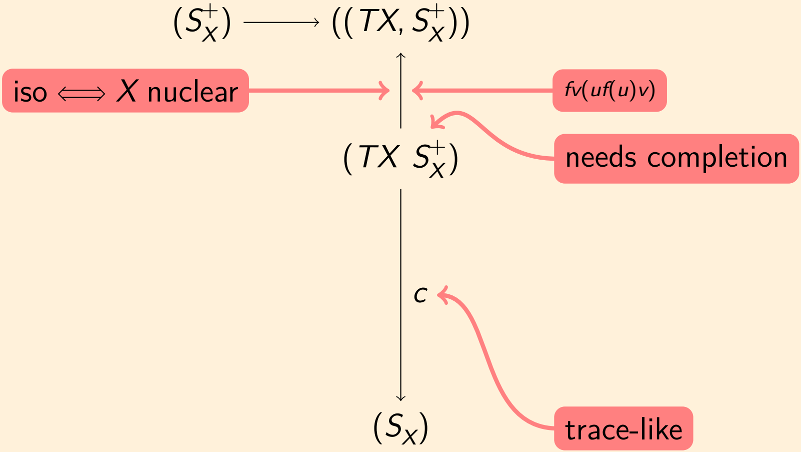
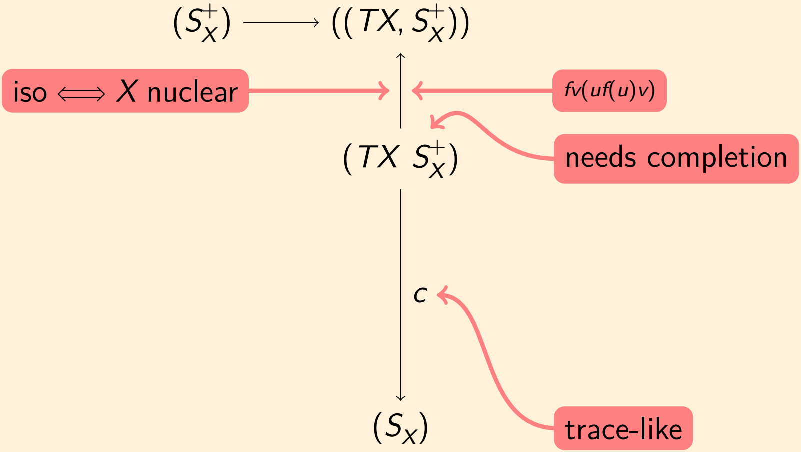
Collapse?



(S_X^+)
$((TX, S_X^+))$
iso $\iff$ X nuclear
$fv(uf(u)v)$
needs completion
$(TX\ S_X^+)$
c
(S_X)
trace-like

Collapse? Not if we're nuclear



Part II

Ingredients Under the Microscope

A Close Examination of the Ingredients

Ingredients

Under the Microscope

Remarks

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Under the Microscope

Construction of S starts from

$$\mathrm{Cl}(W) := T(w) \big/_{w \; w \; g(w, w)1}$$

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So need g on V not V

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Build from (V, g)

Has Clifford Multiplication

$$c \quad V \quad S^{+} \quad S$$

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So need g on V not V

Ratatouille

Gusteau:

"*Anyone can cook*"

or

Anton Ego:

"*No, I don't think anyone can*"

Weak or Strong?

Then the guard looks in politely and will ask you very brightly
“Do you like your morning tea weak or strong?”
But Skimble’s just behind him and was ready to remind him,
For Skimble won’t let anything go wrong. *T. S. Eliot*

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Tea Through the Ages

1911: Weak or Strong

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Tea Through the Ages

1911: Weak or Strong

2011: Black, green, chamomile, strawberry, jasmine, Earl Grey, chai, ... (but not Lipton)

Classifying Orthogonal Structures

Weak	g_p E_p E_p	inner product space
Strong	g_p E_p E_p	Hilbert space

Classifying Orthogonal Structures

Weak	$g_p \;\; E_p \;\; \overline{E}_p \;\; E_p$	fibrewise Hilbert completion
Strong	$g_p \;\; E_p \quad E_p$	Hilbert space

Classifying Orthogonal Structures

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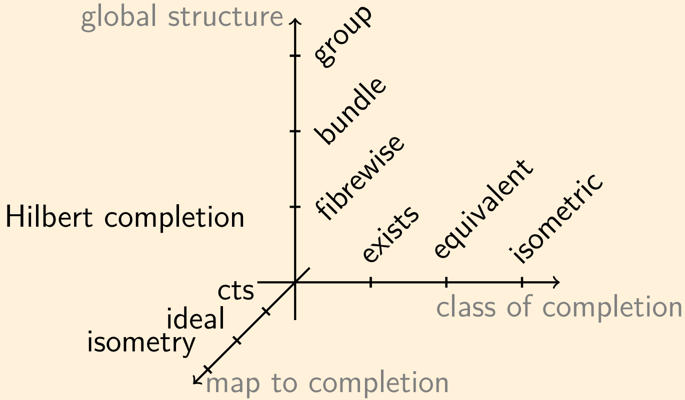


Grow Your Own

Loop Spaces: Common-or-Garden or Organic?

Question

1. Do loop spaces have orthogonal structures on their cotangent bundles?
2. If so, how good?

Loop Spaces: Common-or-Garden or Organic?

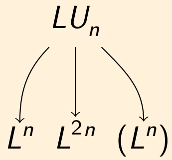
Question

1. Do loop spaces have orthogonal structures on their cotangent bundles?
2. If so, how good?

Answer

1. Yes
2. Good, but not quite as good as on the tangent bundle.

Grow Your Own Orthogonal Structure



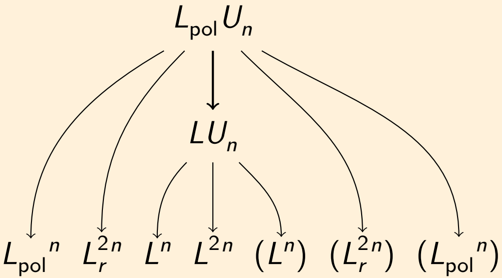
In algebraic topological soil

Grow Your Own Orthogonal Structure

$$
\begin{array}{c}
L_{\text{pol}} U_n \\
\downarrow \\
L U_n \\
\swarrow \quad \downarrow \quad \searrow \\
L^n \quad L^{2n} \quad (L^n)
\end{array}
$$

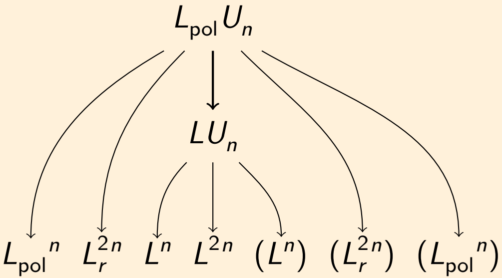
In algebraic topological soil

Grow Your Own Orthogonal Structure



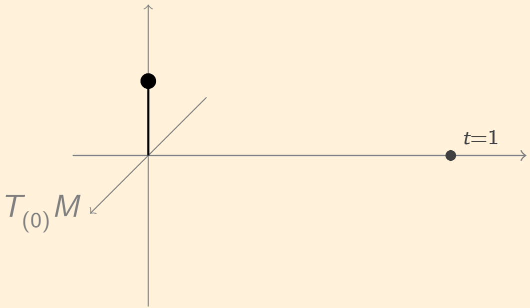
In algebraic topological soil

Grow Your Own Orthogonal Structure



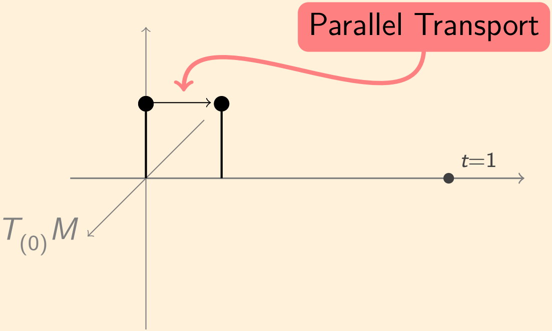
In algebraic topological soil

Grow Your Own Orthogonal Structure



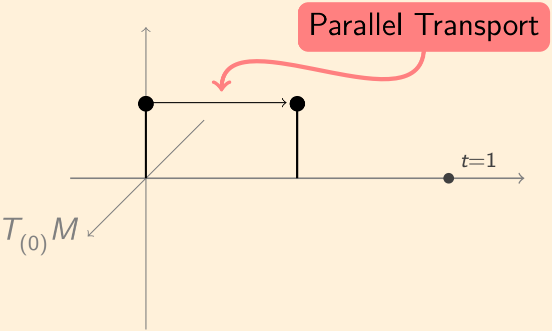
In differential soil

Grow Your Own Orthogonal Structure



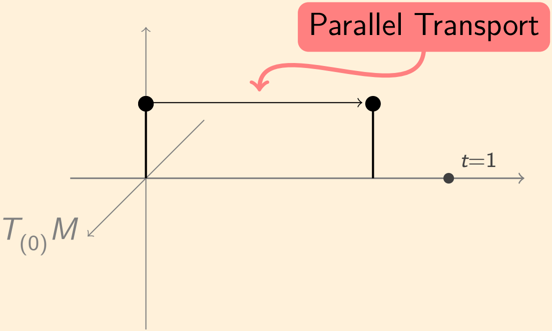
In differential soil

Grow Your Own Orthogonal Structure



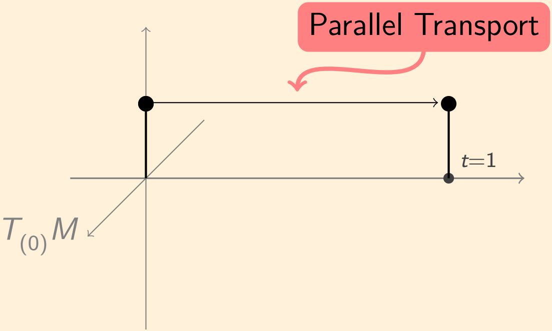
In differential soil

Grow Your Own Orthogonal Structure



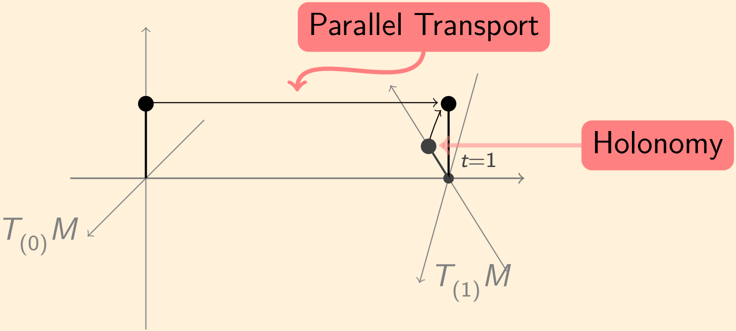
In differential soil

Grow Your Own Orthogonal Structure



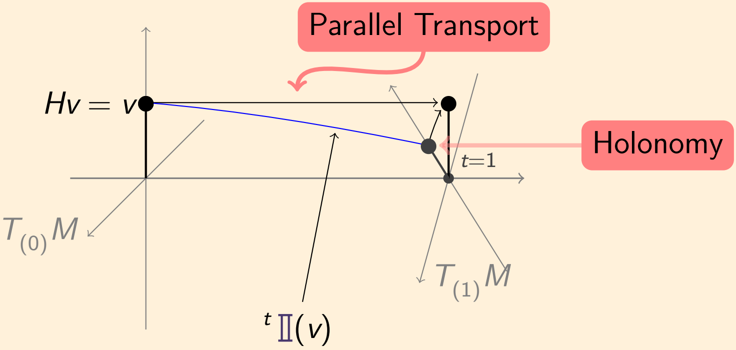
In differential soil

Grow Your Own Orthogonal Structure



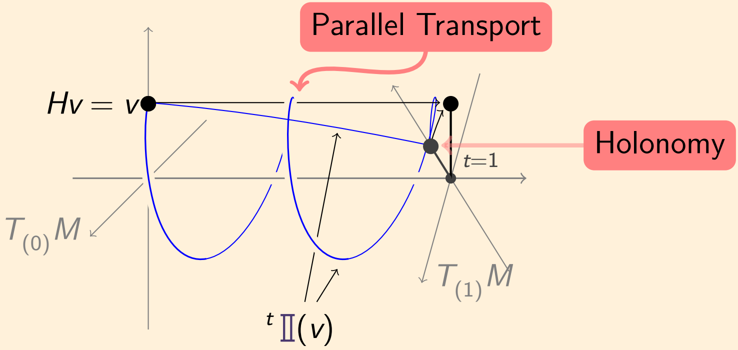
In differential soil

Grow Your Own Orthogonal Structure



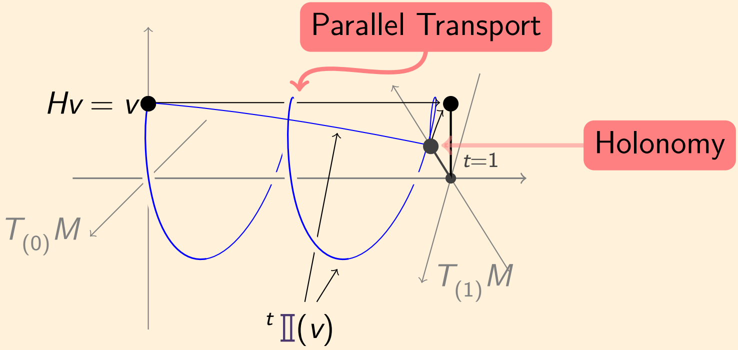
In differential soil

Grow Your Own Orthogonal Structure



In differential soil

Grow Your Own Orthogonal Structure



Any two choices differ by a polynomial

In differential soil

The Best Recipe

Ingredients

Remarks

Method

The Best Recipe

Ingredients

1. Smooth manifold, M
2. Riemannian structure, g

Remarks

oriented, spin, string, even dimensional, …

Riemannian structure on M

Method

1. Add g to M and leave until a connection appears.

The Best Recipe

Ingredients

1. Smooth manifold, M
2. Riemannian structure, g

Remarks

$$g_p \quad T_pM \quad T_pM$$
$$g_p \quad T_pM \quad\quad T_pM$$

Method

1. Add g to M and leave until a connection appears.
2. Apply to TM to produce parallel transport $⟦$.

The Best Recipe

Ingredients

1. Smooth manifold, M
2. Riemannian structure, g

Remarks

Method

1. Add g to M and leave until a connection appears.
2. Apply to TM to produce parallel transport $\mathbb{I}$.
3. Use $\mathbb{I}$ to extract $L_{\text{pol}} TM$ from LTM.

The Best Recipe

Ingredients

1. Smooth manifold, M
2. Riemannian structure, g

Remarks

Method

1. Add g to M and leave until a connection appears.
2. Apply to TM to produce parallel transport $⫴$.
3. Use $⫴$ to extract $L_{\text{pol}}TM$ from LTM.
4. Add to LTM to get $(L_r^2 TM)$.

The Best Recipe

Ingredients

1. Smooth manifold, M
2. Riemannian structure, g
3. Spin structure, Q

Remarks

S^1	$L\,SO_n$	$L\,SO_n$
S^1	Q	P

Method

1. Add g to M and leave until a connection appears.
2. Apply to TM to produce parallel transport $\mathbb{I}$.
3. Use $\mathbb{I}$ to extract $L_{\text{pol}}TM$ from LTM.
4. Add to LTM to get $(L^2_r TM)$.

The Best Recipe

Ingredients

1. Smooth manifold, M
2. Riemannian structure, g
3. Spin structure, Q

Remarks

Method

1. Add g to M and leave until a connection appears.
2. Apply to TM to produce parallel transport $\mathbb{I}$.
3. Use $\mathbb{I}$ to extract $L_{\mathrm{pol}} TM$ from LTM.
4. Add to LTM to get $(L^2_r TM)$.
5. Place Q over the mixture and allow L to infuse upwards.

The Best Recipe

Ingredients

1. Smooth manifold, M
2. Riemannian structure, g
3. Spin structure, Q
4. Spin representations, S

Remarks

Has Clifford Multiplication

$c \; L^n \; S^+ \; S$ (bilinear)

Method

1. Add g to M and leave until a connection appears.
2. Apply to TM to produce parallel transport $\mathbb{I}$.
3. Use $\mathbb{I}$ to extract $L_{\mathrm{pol}} TM$ from LTM.
4. Add to LTM to get $(L^2_r TM)$.
5. Place Q over the mixture and allow L to infuse upwards.

The Best Recipe

Ingredients

1. Smooth manifold, M
2. Riemannian structure, g
3. Spin structure, Q
4. Spin representations, S

Remarks

Clifford Multiplication
becomes
$c\ \ TLM\ \ S^{+}_{LM}\ \ S_{LM}$
(fibrewise bilinear)

Method

1. Add g to M and leave until a connection appears.
2. Apply to TM to produce parallel transport $\mathbb{I}$.
3. Use $\mathbb{I}$ to extract $L_{\mathrm{pol}}TM$ from LTM.
4. Add to LTM to get $(L^2_r TM)$.
5. Place Q over the mixture and allow L to infuse upwards.
6. Combine S with Q to produce bundles S_{LM}.

The Best Recipe

Ingredients

1. Smooth manifold, M
2. Riemannian structure, g
3. Spin structure, Q
4. Spin representations, S

Remarks

Method

1. Add g to M and leave until a connection appears.
2. Apply to TM to produce parallel transport $⫴$.
3. Use $⫴$ to extract $L_{\text{pol}}TM$ from LTM.
4. Add to LTM to get $(L_r^2 TM)$.
5. Place Q over the mixture and allow L to infuse upwards.
6. Combine S with Q to produce bundles S_{LM}.
7. Apply L to S_{LM}^+ to produce a covariant differential operator .

The Best Recipe

Ingredients

1. Smooth manifold, M
2. Riemannian structure, g
3. Spin structure, Q
4. Spin representations, S

Remarks

$$/ \; (S^+_{LM}) \quad ((TLM, S^+_{LM}))$$

$$(TLMS^+_{LM})$$

$$^c \; (S_{LM})$$

Method

1. Add g to M and leave until a connection appears.
2. Apply to TM to produce parallel transport $\mathbb{I}$.
3. Use $\mathbb{I}$ to extract $L_{\text{pol}} TM$ from LTM.
4. Add to LTM to get $(L^2_r TM)$.
5. Place Q over the mixture and allow L to infuse upwards.
6. Combine S with Q to produce bundles S_{LM}.
7. Apply L to S^+_{LM} to produce a covariant differential operator .
8. Combine and c to produce the Dirac operator $/$.

A perfect Dirac soufflé.