Professor Dirac's Cookbook

or

How to Construct a Dirac Operator in Infinite Dimensions

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In Today's Program

Recipe of the Day

Cooking up a Dirac operator

Ingredients Under the Microscope

Orthogonal structures

Grow Your Own

co-Riemannian structure

Part I

Recipe of the Day

The Basic Recipe

Ingredients

1. Smooth manifold, M
2. Riemannian structure, g
3. Spin structure, Q
4. Spin representations, S

Remarks

Method

1. Add g to M and leave until a connection appears.
2. Place Q over the mixture and allow to infuse upwards.
3. Combine S with Q to produce bundles S_M.
4. Apply to S_M^+ to produce a covariant differential operator .
5. Combine , g, and c to produce the Dirac operator $/$.

First Variation

Ingredients

1. Smooth manifold, X
2. Riemannian structure, g
3. Spin structure, Q
4. Spin representations, S

Remarks

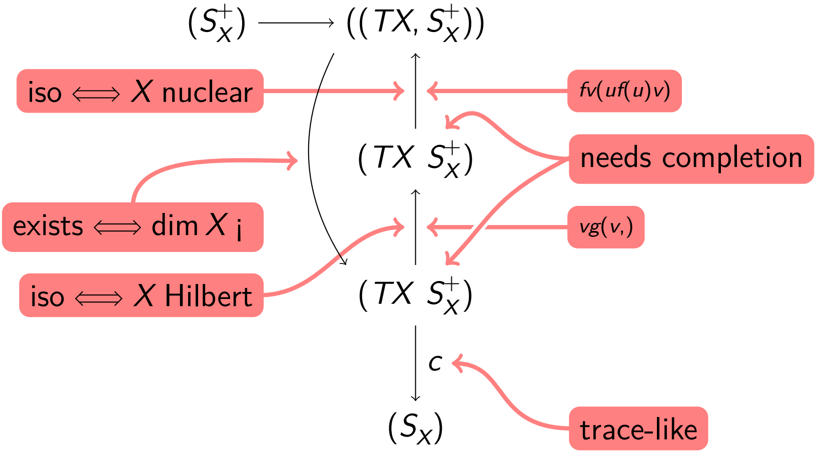
Method

1. Add g to X and leave until a connection appears.
2. Place Q over the mixture and allow to infuse upwards.
3. Combine S with Q to produce bundles S_X.
4. Apply to S_X^+ to produce a covariant differential operator .
5. Combine , g, and c to produce the Dirac operator $/$.

Result

Total collapse of Dirac soufflé.

The Collapse



The Improved Basic Recipe

Ingredients

1. Smooth manifold, M
2. Riemannian structure, g
3. Spin structure, Q
4. Spin representations, S

Remarks

Method

1. Add g to M and leave until a connection appears.
2. Place Q over the mixture and allow to infuse upwards.
3. Combine S with Q to produce bundles S_M.
4. Apply to S_M^+ to produce a covariant differential operator .
5. Combine and c to produce the Dirac operator $/$.

Second Variation

Ingredients

1. Smooth manifold, X
2. Riemannian structure, g
3. Spin structure, Q
4. Spin representations, S

Remarks

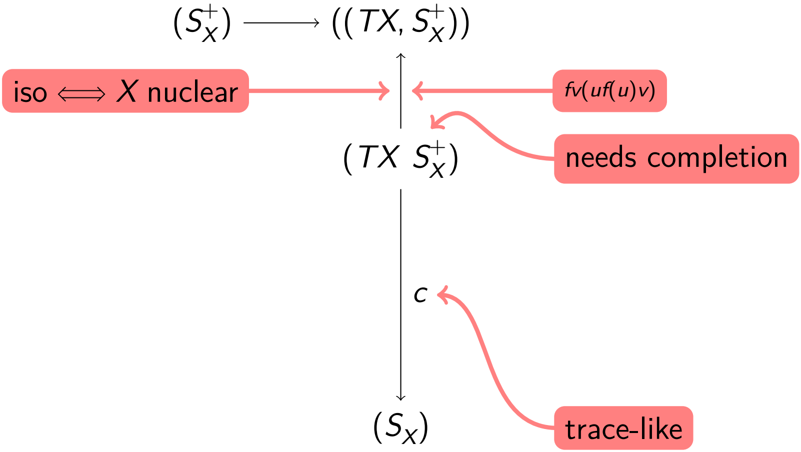
Method

1. Add g to X and leave until a connection appears.
2. Place Q over the mixture and allow to infuse upwards.
3. Combine S with Q to produce bundles S_X.
4. Apply to S_X^+ to produce a covariant differential operator .
5. Combine and c to produce the Dirac operator $/$.

Question

Does it collapse?

Collapse? Not if we're nuclear



Part II

Ingredients Under the Microscope

A Close Examination of the Ingredients

Ingredients

1. Smooth manifold, X
2. co-Riemannian structure, g
3. Spin structure, Q
4. Spin representations, S

Remarks

Build from (V, g)

Has Clifford Multiplication

$$c \; V \; S^{+} \;\; S$$

$$c \;\; TX \; S_{X}^{+} \;\; S_{X}$$

Under the Microscope

Construction of S starts from

$$\mathrm{Cl}(W) := T(w) \big/_{w \;\; w \;\; g(w,w)1}$$

So need g on V not V

Ratatouille

Gusteau:
"*Anyone can cook*"
or
Anton Ego:
"*No, I don't think anyone can*"

Weak or Strong?

Then the guard looks in politely and will ask you very brightly
"Do you like your morning tea weak or strong?"
But Skimble's just behind him and was ready to remind him,
For Skimble won't let anything go wrong. *T. S. Eliot*

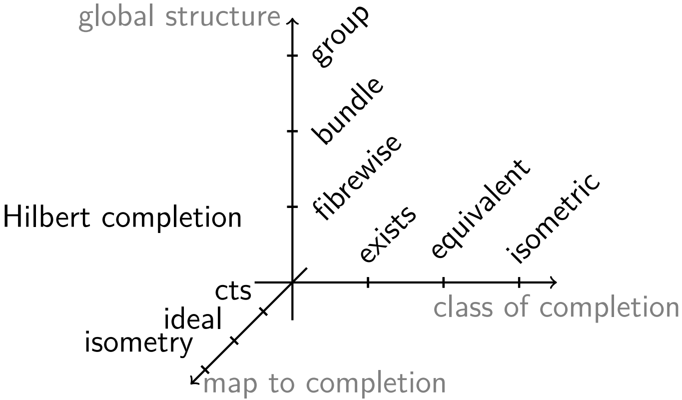
Tea Through the Ages

1911: Weak or Strong

2011: Black, green, chamomile, strawberry, jasmine, Earl Grey, chai, ... (but not Lipton)

Classifying Orthogonal Structures

Weak	g_p E_p E_p	inner product space
Strong	g_p E_p E_p	Hilbert space



Part III

Grow Your Own

Loop Spaces: Common-or-Garden or Organic?

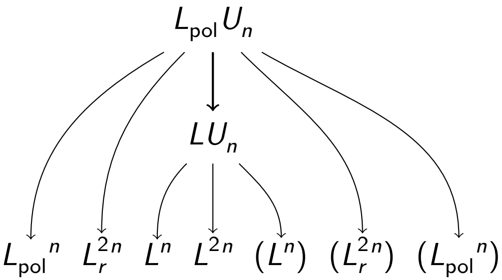
Question

1. Do loop spaces have orthogonal structures on their cotangent bundles?
2. If so, how good?

Answer

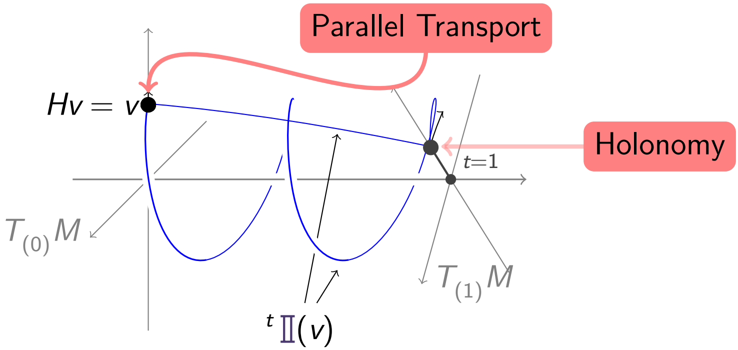
1. Yes
2. Good, but not quite as good as on the tangent bundle.

Grow Your Own Orthogonal Structure



In algebraic topological soil

Grow Your Own Orthogonal Structure



Any two choices differ by a polynomial

In differential soil

The Best Recipe

Ingredients

1. Smooth manifold, M
2. Riemannian structure, g
3. Spin structure, Q
4. Spin representations, S

Remarks

Clifford Multiplication
becomes
$c \ \ TLM \ \ S^{+}_{LM} \ \ S_{LM}$
(fibrewise bilinear)

Method

1. Add g to M and leave until a connection appears.
2. Apply to TM to produce parallel transport $\mathbb{I}$.
3. Use $\mathbb{I}$ to extract $L_{\text{pol}}TM$ from LTM.
4. Add to LTM to get $(L^2_r TM)$.
5. Place Q over the mixture and allow L to infuse upwards.
6. Combine S with Q to produce bundles S_{LM}.
7. Apply L to S^{+}_{LM} to produce a covariant differential operator .
8. Combine and c to produce the Dirac operator $/$.

Result

A perfect Dirac soufflé.