Describing Unstable Operations

Geometry and Topology Seminar, Glasgow

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The Structure

1. $E^*(\underline{E}_k)$ is a graded algebra
2. Co-addition: $\Delta^+ : E^*(\underline{E}_k) \to E^*(\underline{E}_k)\widetilde{\otimes}E^*(\underline{E}_k)$
3. Co-multiplication: $\Delta^\times : E^*(\underline{E}_{k+l}) \to E^*(\underline{E}_k)\widetilde{\otimes}E^*(\underline{E}_l)$
4. Co-linear: $\epsilon^v : E^*(\underline{E}_k) \to E^*(\mathrm{pt}) = E^*$
5. Composition: $E^k(\underline{E}_l) \times E^l(\underline{E}_m) \to E^k(\underline{E}_m)$

Enriched Hopf Ring Structure

1. $E_*(\underline{E}_k)$ is a graded E^*-coalgebra

2. $*$-multiplication $E_*(\underline{E}_k) \otimes E_*(\underline{E}_k) \to E_*(\underline{E}_k)$

3. $\circ$-multiplication $E_*(\underline{E}_k) \otimes E_*(\underline{E}_l) \to E_*(\underline{E}_{k+l})$

4. extra linear structure, $v \in E^k$
   $(\xi v)_* \colon E^* = E_*(\mathrm{pt}) \to E_*(\underline{E}_k)$

5. co-composition ("mposition"), $r \in E^l(\underline{E}_k)$
   $r_* \colon E_*(\underline{E}_k) \to E_*(\underline{E}_l)$

Push Forward Formulae

Example: $K(1)^0(\underline{K(1)}_0)$

$*$-generators: b^J, $J = (j_0, j_1, \dots)$ with $0 \le j_i \le p-1$, almost all zero, and $\sum j_i = 0 \mod p-1$

relations: $(b^J)^{*p} = 0$

$r_* b^J$: b^J built by $\circ$-multiplication of elements b_k; $r_* b_k$ is the coefficient of x^k in the formal identity:

$$r_* b(x) = [\langle r, 1_2 \rangle] * \bigstar_{j=1}^{\infty} b(x)^{\circ j} \circ [\langle r, b_j \rangle]$$

After which, we use the formulae:

$$r_*(a \circ c) = \sum_i \sum_j \pm \bigstar_\alpha r'_{\alpha *} a_{i,\alpha} \circ r''_{\alpha *} c_{j,\alpha}$$

$$r_*(a * c) = \sum_i \sum_j \pm \bigstar_\alpha r'_{\alpha *} a_{i,\alpha} \circ r'''_{\alpha *} c_{j,\alpha}$$

Graded Plethories

1. P^*_k is a graded algebra
2. Co-addition: $\Delta^+ : P^*_k \to P^*_k \widetilde{\otimes} P^*_k$
3. Co-multiplication: $\Delta^\times : P^*_{k+l} \to P^*_k \widetilde{\otimes} P^*_l$
4. Co-linear: $\epsilon^\lambda : P^*_k \to k^*$
5. Composition: $P^k_l \odot P^l_m \to P^k_m$