From Factory to City

How much “stuff” goes from Factory 1 to Trondheim?

\[
\begin{bmatrix}
F_1 & F_2 \\
T & .229 & .235 \\
B & .300 & .300 \\
O & .471 &
\end{bmatrix}
= \begin{bmatrix}
.22 & .25 & .27 & .21 \\
.30 & .30 & .30 & .30 \\
.48 & .45 & .43 & .49 \\
\end{bmatrix}
\begin{bmatrix}
.3 \\
.1 \\
.2 \\
.4 \\
\end{bmatrix}
\]

\[
.48 \times .3 + .45 \times .1 + .43 \times .2 + .49 \times .4 = .4671
\]
Matrices and Column Vectors

\[
\begin{bmatrix}
1 \\
-10
\end{bmatrix}
= 
\begin{bmatrix}
1 & 3 & -4 & 5 \\
2 & 8 & 0 & 0 \\
2 & 1 & -3 & 4
\end{bmatrix}
\begin{bmatrix}
3 \\
-2 \\
-1 \\
0
\end{bmatrix}
\]
A Convenient Notation

Write generic linear system as:

\[ Ax = b \]

Composition order:

- \( A \) and \( B \) represent processes
- feed outputs of \( A \) into \( B \) to get new process, matrix \( C \)
- write \( C = BA \)
- substitute into notation for linear system:

\[ Cx = BAx \]

do \( A \) first and then \( B \)

\[ f(x) = \sin(x) \]

\[ \cos(\sin(x)) \]
The Rules

Recall

- $\mathbb{R}$ has $+$, $\times$, $\div$, $-$, 0, 1 subject to some rules
- $\mathbb{C}$ has $+$, $\times$, $\div$, $-$, 0, 1 subject to same rules

Matrices have $+$, $\times$, $\left(\div\right)$, $-$, 0, (1?)

Question

What rules do these satisfy?