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Special Issue

The Use of a 12-run Plackett–Burman Design in the Injection Moulding of a Technical Plastic Component

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During injection moulding of technical plastic components it is often the case that 15–20 variables need to be set to operational conditions when the production of a new plastic components is started. In this case study, a 12-run Plackett–Burman design is used in the experimentation with eight factors and nine responses to obtain better operational conditions for the production of a certain product. A 12-run Plackett–Burman design belongs to the class of non-geometric orthogonal arrays. Most of these designs have very good projection properties compared with their run sizes, but due to partial aliasing among effects, standard methods of analysing fractional factorial two-level designs do not apply. In this paper, part of the planning of the experiment, the method of analysis used and the results achieved are presented. Substantial improvement in the operational conditions was obtained. Copyright © 2006 John Wiley & Sons, Ltd.

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1. *INTRODUCTION*

This paper reports the results from a 12-run Plackett–Burman (PB) design (Plackett and Burman[1]) performed at Microplast AS, one of the leading companies in injection moulding of technical plastic components in Scandinavia. The work carried out is also part of a project administered by SINTEF, an industrial and technological research institution connected to The Norwegian University of Science and Technology, and supported by the Norwegian Research Council in order to bring technological knowledge from research institutions to small and medium sized companies.

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An injection moulding machine may have 15–20 variables that need to be set to operational conditions wh the production of a product is started. The settings of these variables influence, for example, cycle time ai product quality, thereby greatly affecting the profitability of a company's production. At Microplast AS, findii the specific settings for a new product is done using test-runs, one-factor-at-a-time experimentation togeth with the company's knowledge of its processes and materials. These specific settings are then often used production throughout a product's lifetime.

A driving force behind the project at the company was to improve the cost-efficiency of the injection mouldii through reducing uncertainty in the operational conditions. For instance, could the cycle time, the time from tl start of the process until a product is made, be reduced without sacrificing the quality of the product?

Two meetings were arranged and attended by both leaders and operators. The first was of motivational natu where we explained the efficiency of performing two-level experiments and varying factors simultaneous An agreement to perform an experiment was reached and the company was asked to choose a product and factc that possibly influenced the responses for investigation. For the second meeting the company had chosen component already in production. The experiments to run, number of factors, their levels and other arrangemer in order carry out the investigation were also decided in this meeting.

In Section 2, we describe the factors, responses and experimental design used for investigation. The analys of the data is presented in Section 3. A summary of the results obtained is presented in Section Concluding remarks are given in Section 5.

2. *FACTORS, RESPONSES AND DESIGN*

The cycle time was the main response of interest for the company since lowering this value would enab them to increase production. Hence, the factors chosen for the experiment were factors they believed cou influence the cycle time. In addition, the product under investigation had to meet certain specifications related product quality, which led to the need for measuring other responses. The company came up with eight factc that they assumed would affect the cycle time. These eight factors represented temperature, velocity, time ai pressure. Altogether, eight responses were measured and one response was derived from two of the othei Due to the confidentiality agreement, factors will be denoted as A, B, ..., H and responses as R_1, R_2, ..., R Response R_1 is the cycle time, the others represent several product quality measures.

Some of the factors were expected to have rather large effects on the cycle time, the effects of others we more uncertain, but are still an open question for investigation. Since rather little experimentation had bee performed, we found it natural to perform a screening of the factors and use a fractionated two-level desig The success of such a screening will often rely on a hypothesis of factor sparsity, i.e. only a few factors a responsible for most of the variation in the data. Typically, it is expected that less than half of the facto are important and Box and Meyer[2] consider 0.25 to be a sensible value for the prior probability that a factor active. Hence, in our situation the number of factors influencing a given response is not expected to exceed thre This way of thinking is also in accordance with the principle of parsimony or in Juran's[3] words distinguishii the 'vital few' factors from the trivial many.

Active subspaces of factors may be identified using the projective properties of statistical design. The conce of projectivity was introduced by Box and Tyssedal[4]. An $N \times k$ design with N runs and k factors each at tv levels is said to be of projectivity P if every subset of P factors out of a possible k contains a complete 2 factorial design, possibly with some points replicated. A projectivity P design ensures that all of the effects a model with P factors can be estimated. If the design in addition has replicated points, factor spaces with factors can be investigated with rather weak assumptions on the underlying model since replicated points ha the same expected values regardless of the underlying model.

A very popular two-level design for investigating eight factors under the assumption of factor sparsity is tl 16-run 2^{8-4}_{IV} design (Box *et al.*[5], p. 264). For instance, if only three factors are active, this design projects into fully replicated 2^3 design in any three factors. Also, all main effects can be estimated free of any aliasing wi two-factor interactions. Its disadvantage is that being of resolution IV, two-factor interactions are fully alias

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Table I. 12-run PB design with four centre runs for the injection moulding experiment. Factors J–L are dummy factors

A	B	C	D	E	F	G	H	J	K	L	R_1
1	−1	1	−1	−1	−1	1	1	1	−1	1	15.4
1	1	−1	1	−1	−1	−1	1	1	1	−1	17.3
−1	1	1	−1	1	−1	−1	−1	1	1	1	19.3
1	−1	1	1	−1	1	−1	−1	−1	1	1	17.4
1	1	−1	1	1	−1	1	−1	−1	−1	1	21.3
1	1	1	−1	1	1	−1	1	−1	−1	−1	19.3
−1	1	1	1	−1	1	1	−1	1	−1	−1	17.3
−1	−1	1	1	1	−1	1	1	−1	1	−1	21.4
−1	−1	−1	1	1	1	−1	1	1	−1	1	21.3
1	−1	−1	−1	1	1	1	−1	1	1	−1	19.4
−1	1	−1	−1	−1	1	1	1	−1	1	1	15.3
−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	15.3
0	0	0	0	0	0	0	0	0	0	0	18.4
0	0	0	0	0	0	0	0	0	0	0	18.3
0	0	0	0	0	0	0	0	0	0	0	18.3
0	0	0	0	0	0	0	0	0	0	0	18.4

with other two-factor interactions. This complicates the allocation of active contrasts to the individual effects. Another possibility is the 12-run PB design (Plackett and Burman[1]). This design belongs to the class of non-geometric orthogonal two-level arrays. Designs in this class can not be written as ordinary fractions of the 2^k factorial designs which sometimes are called geometric designs or designs belonging to the 2^{k-p} family; that is, a $1/2^p$ fraction of a 2^k factorial. The advantage of the non-geometric designs are their projective properties and also that they exist when the number of runs, N, is a multiple of four. Most of them will be projectivity $P \geq 3$ designs in $N - 1$ factors while geometric designs can only be used to screen $N/2$ factors at projectivity $P = 3$. For more details about projectivity of two-level designs we refer to Box and Tyssedal[4].

The 12-run PB design projects into a 2^3 design in any three out of 11 factors, with four runs replicated (Lin and Draper[6], Box and Tyssedal[4]). Hence, both the 2_{IV}^{8-4} design and the 12-run PB design allow the estimation of all main effects and interactions for any three factors free of aliasing if other factors can be assumed inert, but for the 12-run PB design, two-factor interactions are only partially aliased with other effects. This may simplify the identification of active factors. Our final decision was then to use the 12-run PB design. Then we could also afford to run four centre runs and thereby obtain a model independent estimate of the error and also check for curvature in the responses for the price of running 16 runs. The design used is shown in Table I. The response, R_1, here is the cycle time. The run order was randomized and for each factor setting some test cycles were carried through and thereafter data were collected for 10 production cycles. Two plastic components were made in each production cycle and the responses analysed are the average measures for all components. The whole experiment was performed in 5 hours.

3. *ANALYSIS OF THE DESIGN*

Non-geometric designs have more complicated alias relationships than the geometric designs. For instance, in a 12-run PB design every main effect may be partially aliased with 45 two-factor interactions and a single two-factor interaction will appear in the alias pattern of all main effects not involved with this two-factor interaction. As a result standard methods often used for analysing non-replicated two-level designs such as normal and half-normal plots (Daniel[7]), or more quantitative methods such as Lenth's method (Lenth[8]) may be of little value since these methods are based on being able to separate active contrasts from contrasts estimating only noise.

However, several methods are also available for analysing these designs. These can mainly be classified as effect based or factor based. Effect-based methods aim at identifying significant effects. A model consisting of

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Table II. The result of a projective-based search for cycle time

One active		Two active		Three active	
$\hat{\sigma}$	Factor	$\hat{\sigma}$	Factor	$\hat{\sigma}$	Factor
1.10	E	0.06	D, E	0.04	C, D, E
2.19	D	1.15	A, E	0.05	D, E, J
2.45	B	1.15	C, E	0.05	B, D, E
2.45	A	1.15	E, G	0.05	D, E, L
2.45	C	1.16	B, E	0.06	A, D, E

main effects and interactions is assumed to give an adequate approximation of the response. The principle o effect heredity, i.e. an interaction is excluded from the model unless at least one (weak heredity) or both (stron heredity) of the main effects associated with the interaction are also included, is often a precept. Examples o such methods can be found in Hamada and Wu[9] and Chipman *et al.*[10]. An effect-based method that does nc depend on the heredity principle is given in Tyssedal and Kulahci[11]. Factor-based methods aim at identifyin active factors and they are less dependent on model assumptions, heredity included. Their drawback may be tha if a large amount of noise is present, it may be difficult to discriminate between several candidate sets of activ factors. Examples of such methods are given in Box and Meyer[2], Kulahci and Box[12], Tyssedal and Samset[13] For both methods it is crucial that the sparsity of either effects or factors, respectively, can be assumed.

We chose to use a factor-based method proposed in Tyssedal and Samset[13], which we will call a projective based search. In fact, the analysis showed that three out of the nine responses most likely did not obey even th weak heredity principle. The method exploits the projective properties of the design (Lin and Draper[6], Box an Tyssedal[4]). In particular, the 12-run PB design projects into a 2^1 design replicated six times in any one facto It projects into a 2^2 design replicated three times in any two factors and it projects into a 2^3 design in any thre factors with four runs replicated.

Thereby assuming at most three active factors, we can investigate how well every set of k factors explain th data $1 \leq k \leq 3$, by examining the fit for replicated runs, calculating

$$\hat{\sigma}^2 = \frac{1}{\sum_{i=1}^{s}(r_i - 1)} \sum_{i:\text{replicated}} \sum_{j=1}^{r_i} (y_{ij} - \bar{y}_i)^2$$

where s is the number of runs replicated and r_i is the number of replications for replicated run number i. The se of factors that causes small values of $\hat{\sigma}^2$ are candidates for being declared active. It should be noted, howevei that blindly picking the set of factors that minimizes $\hat{\sigma}^2$ is not advisable. Rather one should, for each valu of k, make a candidate list of those sets of factors that give the smallest value of $\hat{\sigma}^2$. Simulations have show (Grinde[14]) that there is a high probability that the correct set of factors is included in a candidate list containin the top five sets ranked after $\hat{\sigma}^2$ for each k. A longer list should be used for higher variance. Regression analysi and subject matter knowledge can then be used to reduce this list. If ambiguity still exists, follow-up run (Box *et al.*[15]) should be performed. Often the number of candidate sets that actually need further investigatio is small.

Table II shows the results from a projective-based search for cycle time. For each value of k the top fiv models according to smallest value of $\hat{\sigma}$ are ranked. Clearly the factors D and E should be declared active We observe that $\hat{\sigma}$ for the set of factors (C, D, E) is a little smaller than for the set consisting of only D and E A regression analysis with all main effects and interactions among the three factors C, D and E included reveale a potential active three-factor interaction, CDE. The size of this three-factor interaction, however, was very smal compared with the other significant terms and it was considered rather safe to assume that the factors D and I could explain most of the variation in cycle time.

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Table III. How factors influenced the nine responses

Responses	A	B	C	D	E	F	G	H
R_1				*	*			
R_2	*							
R_3	*		*	*				
R_4			*	*				*
R_5	*	*	*					
R_6	*	*	*					
R_7				*		*		*
R_8				*		*		*
R_9	*		*	*				

In the projective-based search presented above we did not assume any functional relationship between the response and the factors. We only based our procedure on that replicated runs will have the same expected value if our candidate set under investigation contains the true active factors. Every candidate set of factors will have the same expected value for the centre runs. Hence, these provide no information about which factors are active and are not used in the search. However, the four additional runs in the centre are still useful for testing out functional relationships, i.e. whether the models are adequately approximated in terms of main effects and interactions or whether there is any curvature in the responses.

4. SUMMARY OF RESULTS

In multi-response cases there will most likely be correlation among responses and the multivariate nature of data could be taken advantage of to gain better understanding of the underlying mechanism and also to estimate the effects with greater precision (Khuri and Cornell[16], p. 252). Our primary goal, however, was to perform a screening and our main response for investigation was cycle time. The main reason for measuring the other responses was to assure that any change of factor settings in order to improve cycle time would not cause the other quality characteristics to drift away from their desired values.

A search for active factors was carried out for all nine responses and the functional relationships for the most plausible sets were tested out using data from all 16 runs. Table III summarizes how the factors most likely influenced the different responses. We note that factor G was declared inert for all responses, and that factor E only affected cycle time. Two pairs of responses R_5, R_6 and R_7, R_8 were observed to have quite similar models. Also R_3 and R_9 shared the same factor space, but the estimated models were quite different in this case.

The estimated models were used in order to find out whether it was possible to obtain better factor settings. With some compromises, we ended up with a recommended setting of factor levels that occasionally were the same as one of the runs performed in the experiment. Table IV shows the average response values obtained using current factor settings, estimated response values for the recommended factor settings together with the averages obtained with the corresponding experimental run. Values that are closer to their desired values are marked with a '*'. In addition to a 16% decrease in cycle time, we also got noticeable improvements for the responses R_2 and R_3. That the other responses are essentially unchanged was no surprise. It reflects that cycle time was the response of greatest interest and experimental factors were chosen according to this. Also, the estimated models for these responses had small effects.

It was no surprise for the company that factors D and E had an effect on the cycle time, but it was kind of a surprise that the effect of all the six other factors more or less could be neglected. This was new and important information for them. At first they believed that the new recommended factor settings might create impurities on the product surface although no impurities were observed when the experiment was performed. Four months later they decided to implement the factor settings. In order not to risk any impurities they modified the recommended settings of factors D and E somewhat, but they still had a substantial reduction in cycle time.

Table IV. Response values for current factor settings, recommended factor settings and the corresponding experimental run

Response	Current factor settings	Recommended factor settings	Corresponding experimental run	Goal
R_1	18.33	15.33*	15.41*	LOW
R_2	43.20	43.62*	43.59*	HIGH
R_3	7.71	5.87*	6.70*	LOW
R_4	146.665	146.627*	146.642*	146.5
R_5	35.512	35.498	35.528*	36.0
R_6	35.605	35.558	35.614*	36.0
R_7	100.712	100.687*	100.684*	100.4
R_8	69.82	69.8	69.799	70
R_9	0.0934	0.0897*	0.0859*	0

5. CONCLUDING REMARKS

We have used a 12-run PB design to investigate how eight factors potentially could make production of technical plastic component more cost effective. The experiments gave a clear answer how cycle time could b reduced, but also the information about what factors could be anticipated to be inert represented a valuable ga in knowledge for the company. They realized that design of experiments could be a very valuable tool for the plastic part manufacturing. The successful outcome is due to the remarkable projective properties of this desig These made it possible to investigate all candidate sets of three or less factors with respect to their ability (explaining the variability in the data with very weak assumptions about the underlying model.

Finally, we remark that most of the non-geometric two-level orthogonal arrays have good projectic properties (Box and Tyssedal[4]), and will be very useful design alternatives when many factors need to b investigated in few runs and factor sparsity can be assumed. More attention should be paid to their use.

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