Løysingsforslag ST1101/ST6200 H-2012

Oppgåve 1

Gitt skuldig: 3 uavh. forsøk

Reg: stemt skuldig / stemt ikkje skuldig

P(stemt skuldig) = 0.7 for alle 3

$\Rightarrow$ X = talet som stemmer skuldig gitt at personen er skuldig

er $B(3, 0.7)$

da $A \sim$ Erklært skuldig

$S \sim$ Personen er skuldig

$U \sim$ Personen er uskuldig

$$P(A/S) = P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.216 = \underline{0.784}$$

da Z = talet som stemmer skuldig gitt at personen er uskuldig

$Z \sim B(3, 0.2)$

$$P(A/U) = P(Z \geq 2) = 1 - P(Z \leq 1) = 1 - 0.896 = \underline{0.104}$$

b) $P(A) = P(A \cap S) + P(A \cap U) = P(A/S) \cdot P(S) + P(A/U) \cdot P(U)$

$$= 0.784 \cdot 0.7 + 0.104 \cdot 0.3 = \underline{0.58}$$

$$P(S/A) = \frac{P(A \cap S)}{P(A)} = \frac{0.784 \cdot 0.7}{0.58} = \underline{0.946}$$

c) $P(E_1) = P(E_1 \cap S) + P(E_1 \cap U) = P(E_1/S) \cdot P(S) + P(E_1/U) \cdot P(U)$

$$= 0.7 \cdot 0.7 + 0.2 \cdot 0.3 = \underline{0.55}$$

$P(E_1 \cap E_2) = P((E_1 \cap E_2) \cap S) + P((E_1 \cap E_2) \cap U) = P((E_1 \cap E_2)/S) \cdot P(S) + P((E_1 \cap E_2)/U) \cdot P(U)$

$= P(E_1/S) \cdot P(E_2/S) \cdot P(S) + P(E_1/U) \cdot P(E_2/U) \cdot P(U) = 0.7 \cdot 0.7 \cdot 0.7 + 0.2 \cdot 0.2 \cdot 0.3 = 0.355$

$P(E_1 \cap E_2) \neq P(E_1) \cdot P(E_2) \Rightarrow E_1$ og E_2 er avhengige

Oppgåve 2

a) $Y \sim N(25, 0.4)$

$$P(24 < Y < 26) = P\left(\frac{24-25}{\sqrt{0.4}} < \frac{Y-25}{\sqrt{0.4}} < \frac{26-25}{\sqrt{0.4}}\right) = \Phi\left(\frac{1}{\sqrt{0.4}}\right) - \Phi\left(-\frac{1}{\sqrt{0.4}}\right)$$

$$= \Phi(1.58) - \Phi(-1.58) = 0.9429 - 0.0571 = \underline{0.886}$$

$$L = \sum_{i=1}^{40} Y_i, \quad E[L] = \sum_{i=1}^{40} E[Y_i] = 40 \cdot 25 = 1000$$

$$Var[L] = \sum_{i=1}^{40} Var[Y_i] = 40 \cdot 0.4 = 16$$

$$P(L > 1008) = P\left(\frac{L-1000}{4} > \frac{1008-1000}{4}\right) = P(Z > 2) = 1 - \Phi(2) = 1 - 0.9772$$
$$= \underline{0.0228}$$

b)

$$L_1 - L_2 \sim N(0, 32)$$

$$P(|L_1 - L_2| < 10) = P(-10 < L_1 - L_2 < 10) = P\left(-\frac{10}{\sqrt{32}} < \frac{L_1 - L_2 - 0}{\sqrt{32}} < \frac{10}{\sqrt{32}}\right)$$

$$= \Phi\left(\frac{10}{\sqrt{32}}\right) - \Phi\left(-\frac{10}{\sqrt{32}}\right) = \Phi(1.77) - \Phi(-1.77) = 0.9616 - 0.0384 = \underline{0.9232}$$

$$P(\min(L_1, L_2) < 1000) = 1 - P(L_1 \geq 1000 \cap L_2 \geq 1000) = 1 - P(L_1 \geq 1000) \cdot P(L_2 \geq 1000)$$
$$= 1 - \frac{1}{2} \cdot \frac{1}{2} = \underline{0.75}$$

Oppgåve 3

a) $t = 24 \Rightarrow E[X] = \lambda t = \frac{15}{24} \cdot 24 = \underline{15}$

$$P(X > 20 \mid t = 24) = 1 - P(X \leq 20 \mid t = 24) = 1 - \sum_{k=0}^{15} \frac{15^k}{k!} e^{-15} = 1 - 0.917 = \underline{0.083}$$

b) $P(T > t \mid t \leq 0) = 1$

$$P(T > t \mid t \geq 0) = P(X = 0 \text{ i } [0, t]) = \frac{(\lambda t)^0 e^{-\lambda t}}{0!} = e^{-\lambda t}$$

$$P(T>2) = e^{-2\lambda} = e^{-\frac{15}{12}} = 0,287$$

c)

$$F_T(t) = P(T \le t) = 1 - P(T > t) = 1 - e^{-\lambda t}$$

$$f_T(t) = \frac{dF_T(t)}{dt} = \lambda e^{-\lambda t}, \quad t \geq 0$$

$$E[T] = \int_0^{\infty} t \lambda e^{-\lambda t} dt = \left[-t e^{-\lambda t}\right]_0^{\infty} + \int_0^{\infty} e^{-\lambda t} dt$$

$$= \left[\frac{1}{\lambda} - e^{-\lambda t}\right]_0^{\infty} = \frac{1}{\lambda}$$

d)

$$f_{X_1}(k) = f_{X_2}(k) = \frac{(24\lambda)^k e^{-24\lambda}}{k!}$$

$$f_{X_3}(k) = \frac{(16\lambda)^k}{k!} e^{-16\lambda}$$

$$L(\lambda) = \prod_{i=1}^{3} f_{X_i}(k_i) = \frac{(24\lambda)^{k_1} e^{-24\lambda}}{k_1!} \cdot \frac{(24\lambda)^{k_2} e^{-24\lambda}}{k_2!} \cdot \frac{(16\lambda)^{k_3} e^{-16\lambda}}{k_3!}$$

$$= \frac{24^{k_1+k_2} \cdot 16^{k_3} \cdot \lambda^{k_1+k_2+k_3} \cdot e^{-64\lambda}}{\prod_{i=1}^{3}(k_i!)}$$

$$\ln L(\lambda) = (k_1+k_2)\ln 24 + k_3 \ln 16 + (k_1+k_2+k_3)\ln\lambda - 64\lambda - \ln \prod_{i=1}^{3}(k_i!).$$

$$\frac{\partial \ln L}{\partial \lambda} = 0 \Leftrightarrow \frac{\sum k_i}{\lambda} = 64 \Rightarrow \lambda_e = \frac{\sum k_i}{64}$$

$$\hat{\lambda} = \sum_{i=1}^{3} \frac{X_i}{64} \qquad E(\hat{\lambda}) = \sum_{i=1}^{3} \frac{E[X_i]}{64} = \frac{24\lambda + 24\lambda + 16\lambda}{64} = \lambda$$

$$Var(\hat{\lambda}) = \frac{1}{64^2} Var \sum_{i=1}^{3} X_i = \frac{1}{64^2} \sum_{i=1}^{3} Var[X_i] = \frac{24\lambda + 24\lambda + 16\lambda}{64^2} = \underline{\frac{\lambda}{64}}$$

d)

La T_i vere tid mellom fødsel $i-1$ og i.

$$Y = \sum_{i=1}^{100} T_i$$, T_i er uavhengige og eksponentialfordelte.

$$\Rightarrow \sum_{i=1}^{100} T_i \text{ er gammafordelt } (100, \lambda)$$

$$E[Y] = \frac{100}{\lambda}, \quad Var[Y] = \frac{100}{\lambda^2} \Rightarrow Y \approx N\left(\frac{100}{\lambda}, \frac{100}{\lambda^2}\right)$$

$$P(Y > 144) = P\left(\frac{Y - \frac{100}{\lambda}}{\sqrt{\frac{100}{\lambda^2}}} > \frac{144 - \frac{100}{\lambda}}{\sqrt{\frac{100}{\lambda^2}}}\right) = P\left(Z > \frac{-16}{\frac{10}{\frac{5}{8}}}\right)$$

$$= 1 - \Phi(-1) = 1 - 0.1587 = \underline{0.8413}$$