Løsningsskisse til ST1101 / ST6200 Sannsynlighetsregning
Torsdag 29. mai 2008

Oppgave 1

a) Uavhengige kast.
X_1=Resultat for første terning, X_2=Resultat for andre terning.
$P(X_1+X_2=7)=P(X_1=1,X_2=6)+P(X_1=2,X_2=5)+\ldots+P(X_1=6,X_2=1)$
$=0,5^2+5\times 0,1^2=\underline{\underline{0,30}}$

b) $P(X_1=X_2)=P(1,1)+P(2,2)+\ldots+P(6,6)=2\times 0,5\times 0,1+4\times 0.1^2=\underline{\underline{0,14}}$
Hendelse *A*: Sekser, og *B*: Første terning. Skal finne $P(B\mid A)$.
$P(B)=0,5$, $P(A\mid B)=0,1$, $P(A)=P(A\mid B)P(B)+P(A\mid\overline{B})P(\overline{B})=0,30$

$$P(B\mid A)=\frac{P(A\mid B)P(B)}{P(A)}=\frac{0,1\times 0,5}{0,3}=\underline{\underline{0,167}}$$

Oppgave 2

X= Antall tester for strategi *B*.

a) Sannsynlighetsfordeling:

x	$P(X=x)$
1	$(1-p)^k$
$1+k$	$1-(1-p)^k$

$$E(X)=(1-p)^k+(1+k)\left[1-(1-p)^k\right]=\underline{\underline{1+k-k(1-p)^k}}$$

b) $f(k)=\underline{\underline{1-\left(\frac{1}{k}\right)^{-k}}}$

La *Y* være antall tester for *n* personer, hvor $n=m\times k$

$$E(Y)=m\times E(X)=\frac{n}{k}\left[1+k-k(1-p)^k\right]=\underline{\underline{n\left[1+\frac{1}{k}-(1-p)^k\right]}}$$

Oppgave 3

X = Forsinkelse.

a) Finn k. $\int_0^\infty kxe^{-2x}dx = k\frac{1}{2^2} = \frac{k}{4} = 1 \Rightarrow \underline{\underline{k=4}}$

$$E(X) = \int_0^\infty xg(x)dx = \int_0^\infty 4x^2e^{-2x}dx = 4\frac{2}{2^3} = \underline{\underline{1}}$$

$$P(X>2) = \int_2^\infty 4xe^{-2x}dx = \ldots \text{ (delvis integrasjon)} = 5e^{-4} \approx \underline{\underline{0,09}}$$

b) V = antall dager forsinket mer enn 2 min. $\sim$ *Binomisk*$(n=22, p=0,09)$
Antagelser: Uavhengige dager og konstant p for alle dagene.

$$P(V\geq 2) = 1-P(V\leq 1) = 1-\left[P(V=0)+P(V=1)\right] = 1-(1-p)^{22} - 22p(1-p)^{21} = \underline{\underline{0,6012}}$$

Bruker normaltilnærmingen:

$$P(V>30) = 1-P(V\leq 30) \approx 1-P\left(Z \leq \frac{30+0.5-220\times 0,09}{\sqrt{220\times 0,09\times(1-0,09)}}\right) = 1-P(Z\leq 2,52) = \underline{\underline{0,0059}}$$

c) Y = Antall minutter på stasjonen.
$x=2 \Rightarrow f(y\mid x=2) = e^{-y}$ for $y>0$, dvs. eksponensialfordelingen med parameter lik 1. $E(Y\mid x=2) = \underline{\underline{1}}$

$$f(x,y) = f(y\mid x)f(x) = \left(\frac{x}{2}\right)e^{-xy/2}4xe^{-2x} = \underline{\underline{2x^2e^{-x(y/2+2)}}} \text{ for } x>0, y>0.$$

$$h(y) = \int_0^\infty f(x,y)dx = 2\int_0^\infty x^2e^{-x(y/2+2)}dx = 2\frac{2}{(y/2+2)^3} = \underline{\underline{32(y+4)^{-3}}} \text{ for } y>0.$$

Oppgave 4

X = Antall kast før første kron, dvs. X er geometrisk fordelt.

$$M_X(t) = E(e^{tX}) = \sum_{k=1}^\infty e^{tk}\left(\frac{1}{2}\right)^k = \sum_{k=1}^\infty\left(\frac{1}{2}e^t\right)^k$$

Har at $\sum_{k=0}^\infty r^k = \frac{1}{1-r}$ for $0<r<1$

$$\Rightarrow M_X(t) = \sum_{k=0}^\infty\left(\frac{1}{2}e^t\right)^k - 1 = \frac{1}{1-\frac{1}{2}e^t} - 1 = \underline{\underline{\frac{e^t}{2-e^t}}}$$

$$M_X^{(1)}(t)=\frac{2e^t}{\left(2-e^t\right)^2},\ M_X^{(2)}(t)=\frac{2e^t\left(2+e^t\right)}{\left(2-e^t\right)^3}$$

$$E(X)=M_X^{(1)}(t=0)=\underline{\underline{2}}$$

$$E\left(X^2\right)=M_X^{(2)}(t=0)=6\Rightarrow Var(X)=E\left(X^2\right)-\left[E(X)\right]^2=6-2^2=\underline{\underline{2}}$$