Service time assessment in Computational Information Fluid Dynamics using level set methods

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Abstract

The Information Fluid Dynamics concept presented in [DV01] brings an elegant framework for heavy traffic system dynamics simulation. For computational performance and quality of service assessment, key performance indicators like service times cannot be missed. In this paper is presented a method that accurately measures the service times. Measurements are performed periodically with arbitrary user-defined sampling rate. For the sake of simplicity, the paper only shows numerical results performed on basic single system component. But this can be easily extended to complex systems with interacting block components like credit-based flow control or "best effort" traffic.

Contents

1 Introduction and objectives

The objectives of this paper is to define a general framework of service time assessment using Computation Information Fluid Dynamics models proposed in [DV01].
Simulation can give enlightenments on how pilot or control a system. It can also be used for optimization and bottleneck prediction. But because today's infrastructures and subsystems grow along many dimensions with heavy traffic connections, one often needs to continuously estimate key performance indicators and in particular service times. This is the context of this paper.

If a Lagrangian approach ([LL96]) is used to describe the system dynamics, the service time assessment needs small computational efforts. Unfortunately, such a method cannot be easily implemented to preserve mass conservation, which can possibly lead to large errors of mass under large times of observation. More, it becomes unresolvable for large scale and heavy traffic flow because of the large amount of particle (event) to follow (handle). Inversely, an Eulerian method brings more easily conservation properties and the numerical complexity can stay under control for large scale heavy traffic flows. The counterpart is the difficulty to easily compute service times because of the lack of description particle per particle. Some additional work has to be performed. The Information Fluid Dynamics framework presented in [DV01] adopts a continuous Eulerian approach for modelling buffer and processing systems.
In the case of piecewise constant departure rates, analytical expressions of service times of queuing networks can be obtained. But when complex policies of flux management like those for credit-based flow control (see [BCKL95]) or for "best effort" traffic context (see [RA95]) are used, departure rates are rarely piecewise constant. This is due to some limiting flux process that adapts the flow to its optimal (maximal) capacity. In this context, one needs numerical tools for approximating service times.

In this paper, we propose an algorithm to periodically measure service times for some arbitrary periods. The idea follows those of level set tracking methods initiated by Osher and Sethian [OS88] and widely used in Computational Fluid Dynamics ([Set99]) for mass front tracking needs. Suppose that we wish to follow a front convected by a continuous fluid of continuous velocity v. Usual applications are flame front tracking in combustion, chemically reacting layer or transition interface in multi-phase flows. In one space dimension, Osher and Sethian's formalism is very simple : let us define a continuous function $\varphi : \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$ such that, at the initial time, $\varphi(x, 0) \neq 0$ except at position $x = 0$ that indicates the initial position of the front. Level function φ is then convected at velocity v according to the partial differential transport equation:

$$\frac{\partial}{\partial t}\varphi \ + \ v(x,t)\,\frac{\partial}{\partial x}\varphi \ = \ 0, \tag{1}$$

equipped with convenient boundary conditions. The equation is read in the weak sense of distributions. In particular, the product $v\partial_x\varphi$ has a sense in $\mathcal{D}'$ because v is supposed to be continuous. It is easy to see that the level set $\{x/\varphi(x,t) = 0\}$ localizes the convected front at time t.

These ideas can be used in our context with slight modifications. Each time we want to estimate a service time, an information particle is "colored" by a level set. Thus, we need to managed a possibly large amount of colored particles with different level sets. We decide to characterize these levels sets by integer values. Of course, the numerical scheme has not to produce spurious numerical integer values which can be argued by some local discrete maximum principle.

Secondly, we possibly need to handle level functions with less regularity. This is due to the kind of application we consider. Indeed, the service time assessment can be performed either manually or automatically. In the automatic case where measurements are wished periodically, one can easily build a space-time continuous level functions that crosses integer values at the good places. In the manual case, where "someone" directly acts on the simulated system because of direct needs of information (accident study, control, supervision), the event sent by the user has to be instantaneously integrated in the shape of the level function, leading to discontinuities that cross integer values. Finally, the model of queue proposed in [DV01] leads to time-discontinuous propagation velocity v because of possible time-discontinuous arrival and departure rates. In the case where both v and φ are discontinuous at the same point, then the nonconservative product $v\partial_x\varphi$ has no more sense in the distributions. Although some authors (Dal Maso, Le Floch and Murat [DMLFM95]) have proposed some definitions in spaces of measures, we rather like to formulate equation (1) in a conservation form because it is possible in our context. The conservation equation of density of information $\rho > 0$ inside the queue

$$\frac{\partial}{\partial t}\rho \ + \ \frac{\partial}{\partial x}(\rho v) \ = \ 0 \tag{2}$$

can be used to derive the conservation equation

$$\frac{\partial}{\partial t}(\rho\varphi) \ + \ \frac{\partial}{\partial x}(\rho v\varphi) \ = \ 0 \tag{3}$$

for smooth solutions. We can then then use equation (3) for any locally bounded variation solutions φ and read it in the sense of distributions.

This paper is organized as follows: in section 2 we precisely define service times and study the case of piecewise constant departure rates. In section 3, we describe the level set formalism and how (3) is obtained. Section 4 exposes the overall algorithm and comments some principles and characteristic time ratios for computational purposes. Section 5 presents the numerical discretization of equation (3) and gives results of local discrete maximum principles. The last sections presents numerical results and proves both feasibility and performance of such a method.

2 The service time in a multi-class FIFO queue

2.1 Definition

Let us first recall that the travelling of an information particle of fluid k inside a FIFO queue with length brought back to one by transformation (see [DV01]) is governed by the conservation law

$$\frac{\partial \rho_k}{\partial t} + \frac{\partial}{\partial x}(\rho_k v) = 0, \tag{4}$$

where the unknown $\rho_k(x,t)$ is the volume density of k fluid at normalized position $x \in]0,1[$ and time t, $v = v(x,t)$ is the propagation velocity inside the queue of expression

$$v(x,t) = \frac{x\Phi_{eo}(t) + (1-x)\phi_{ei}(t)}{s(t)} \tag{5}$$

where s is the queue size, solution of the ordinary differential equation

$$\frac{ds}{dt} \ = \ \Phi_{ei}(t) \ - \ \Phi_{eo}(t), \tag{6}$$

where $\Phi_{ei}(t)$ and $\Phi_{eo}(t)$ are respectively the effective incoming (or arrival) rate and the effective outgoing (departure) rate. Term "effective" means that there possibly exists a limiting rate process due to conditions of availability or load rate in neighboring components or subsystems. For details, the reader can refer to the large literature on this question ([RA95],[ABR98] for example). This is also discussed in [DV01] for numerical purposes of positiveness and range preserving.

If $x(t; t_0, x_0)$ denotes a queue particle at position x_0 at time t_0, the Lagrangian representation of the queue dynamics is given by the motion equation of x:

$$\dot{x}(t; t_0, x_0) \;=\; v\left(x(t; t_0, x_0), t\right). \tag{7}$$

The analytic solution of equation (7) is not known in general under the hypothesis of general time-varying arrival and departure rates.
Thus, the measurable service time $\delta(t)$ at time t for the FIFO device can be expressed by the travel time of particle $x(t; t, 1)$ through the queue. That requires to know a certain history of the queue life, or equivalently a history of both $\Phi_{ei}(t)$ and $\Phi_{eo}(t)$. The service time $\delta(t)$ can also be represented by the implicit equation

$$x(t - \delta(t); t, 1) \;=\; 0,$$

that means that particle $x(t; t, 1)$ enters the queue at position $x = 0$ at time $t - \delta(t)$.

For a Lagrangian method based on the solution of the motion equation (7), the computation of $\delta(t)$ is a direct consequence of the particle travelling information. For an Eulerian method based of balance equations, some additional work has to be performed in the general case for the computation of service time. This is the object of the main section 3 below. First, let us study the simpler case when the total departure rate is constant.

2.2 Analytical solutions for constant total departure rate

Suppose that the departure position of the queue is fixed on a y axis at point $y = 0$ with free boundary arrival position at position $y_0 = -s(t_0)$ at time t_0. Then, if Φ_o is constant, then the motion equation of the particle inside the queue $y(u; t_0, y_0)$ at time u with initial position y_0 at time t_0 is

$$\dot{y}(u; t_0, y_0) \;=\; \Phi_0,$$

so that the service time $\delta(t)$ of particle $y(t;t_0,y_0)$ is clearly

$$\delta(t) \;=\; \frac{s(t_0)}{\Phi_o} \tag{8}$$

measured at time

$$t \;=\; t_0 \;+\; \frac{s(t_0)}{\Phi_o}.$$

This formula can be very useful for performance assessment for isolated components or for configurations where queue bounds are rarely reached. But it becomes ineffective in the fluid context when several components interact because of flux limitations and neighboring dependencies involving limited time-varying departure rates. This is in particular the case for important applications like Kanban processes or "best effort" communication network policies like ATM networks ([BCKL95]). Thus, we generally need a numerical tool for service time assessment. Its construction is presented in what follows.

3 Particle tracking using level functions

3.1 Building of a level function

Let us introduce a characteristic sample time θ and a time-varying increasing, piecewise constant and left continuous level step function $\varphi_{a,\theta} \in [0,+\infty[\to \frac{1}{2}+\mathbb{N}$ defined as

$$\varphi_{a,\theta}(t) \;=\; \frac{1}{2}+n \text{ for } t\in[n\theta,(n+1)\theta[,\; n\in\mathbb{N}. \tag{9}$$

Reference a states for "arrival" because we wish to follow a particle that enters the queue at a particular discrete time t of the form $n\theta$. The jump of $\varphi_{a,\theta}(t)$ from $n-\frac{1}{2}$ to $n+\frac{1}{2}$ signals (in a certain way) the resetting and activation of some chronometer.

To this time-varying function, we associate a space-time function $\varphi : [0,1]\times[0,+\infty[\to [\frac{1}{2},+\infty[$ solution of the following partial differential convection problem

$$\frac{\partial}{\partial t}\varphi \;+\; v(x,t)\,\frac{\partial}{\partial x}\varphi \;=\; 0,\; x\in]0,1[,\; t>0, \tag{10}$$

$$\varphi(x,0) \;=\; -\frac{1}{2}\;\forall x\in[0,1], \tag{11}$$

$$\varphi(0,t) \;=\; \varphi_{a,\theta}(t)\;\forall t>0 \tag{12}$$

Suppose first that effective fluxes are smooth time-varying functions. Then problem (10),(11),(12) is well posed with solution in $L^\infty\left((0,T), BV(]0,1[)\right)$ for any $T>0$ as long as $v(x,t)$ stays positive. This is indeed the case if the flux limiting process on the effective departure rate $\Phi_{eo}(t)$ ensures $s(t)$ to stay strictly positive. But in realistic cases, $\Phi_{ei}(t)$ and $\Phi_{eo}(t)$ are quite irregular, see discontinuous. In this last case, the nonconservative product

$$v(x,t)\,\frac{\partial\varphi}{\partial x} \tag{13}$$

are no more correct definition in the classical sense of distributions if both v and ρ_k present discontinuities at the same point. Of course, we could use the nonconservative product definition of Dal Maso, Le Floch and Murat [DMLFM95] by adding a family of constrained Lipschitz continuous paths connecting states to the expression (13), giving it a definition in a space of bounded Borel measures. But at the present time, the numerical approximation by difference methods is still an opened problem.
Here we rather proceed by reformulating equation (10) into some conservative equations. We can formally multiply equation (4) by φ and equation (10) by ρ_k. Summing up the two equations leads to the conservative equation

$$\frac{\partial}{\partial t}(\rho_k\varphi) \;+\; \frac{\partial}{\partial x}\left(\rho_k\varphi v\right) \;=\; 0, x\in]0,1[,\; t>0. \tag{14}$$

It is easy to show that problem (14),(11),(12) is well posed with solution belonging to $L^\infty\left((0,T), BV(]0,1[)\right)$ if Φ_{ei} or Φ_{eo} belong to $L^\infty\cap BV(0,T)$ for any T.

Because all the fluid component k share the same propagation velocity, summing up the ks gives the additional partial differential equation

$$\frac{\partial}{\partial t}(\rho\varphi) \;+\; \frac{\partial}{\partial x}\left(\rho\varphi v\right) \;=\; 0, x\in]0,1[,\; t>0. \tag{15}$$

where $\rho=\sum_k \rho_k$ is the total density. It has been proved in [DV01] that ρ does not depend on x for the FIFO model and is exactly equal to $s(t)$. Nevertheless, for usual notation, we keep the notation ρ.
The levels of solution φ follow the trajectories of information particles because all share the same field of velocity and are convected in the same way.

3.2 Some properties of the solution

Notice that at the initial time $t = 0$, the solution φ is constant, equal to $-\frac{1}{2}$. For small positive time, the solution φ at fixed t is made of a step that propagates to the right. while this step has not reached position $x = 1$, we cannot evaluate service times accurately because we suffer from a lack of past information that is not completely included in the initial data.
When the step reaches $x = 1$ at time $t^\star$, the solution instantaneously jumps from value $\varphi = -\frac{1}{2}$ to value $\varphi = \frac{1}{2}$ crossing the integer value 0. Time $t^\star$ is exactly $\delta(t^\star)$ for particle $x(t^\star; 0, 0)$. We have reproduced a zero-crossing event sending that are commonly used in component-based integrated development environments (IDE) like Matlab/Simulink [Inc96].
For larger times t, an event is sent for service time measurement each time an integer k is "crossed" by the level function $\varphi(1,t)$. The service time associated to the k-th event is then given by

$$\delta^k(t) = t - k\theta > 0. \tag{16}$$

This is a generalized version of the zero-crossing function event sending.

From this construction, it is easy to define an algorithm for service time assessment. This is resumed in the next section.

4 Presentation of the algorithm

4.1 Algorithm

Initialization. Choose θ, initialize the field as prescribed by (11), choose a final time T;
set $current\_time = 0$;
set $current\_level = -1$;

For t from 0 to T do {

Choose a "small" time step τ^n;
solve ODE-PDE problem between $current\_time$ and $current\_time + \tau^n$;

refresh $current\_time := current\_time + \tau^n$;

if $\varphi(1, current\_time) \neq current\_level$ {

for each integer k in $]current\_level, \varphi(1, current\_time)[$ {

set $\delta^k(current\_time) = current\_time - k\,\theta$;

}

refresh $current\_level := \varphi(1, current\_time)$;

}

}

4.2 Comments. Ratio of characteristic times

In the algorithm, we need to choose a small time step; "small" refers to the fact that when the algorithm is used for computational purposes, τ^n plays the role of the discretization time step. But there are also two other characteristic times, namely the characteristic sample time θ for which "events" are sent for service time measurements, and the characteristic queue travel time $\tau_t(t)$ depending on the queue size s and both departure and arrival rates according to (5). Under the hypothesis where Φ_{ei} and Φ_{eo} are constant equal to Φ_e with s constant equal to s_0, the characteristic travel time is

$$\tau_t(t) \;=\; v \;=\; \frac{\Phi_e}{s_0}.$$

- If $\frac{\tau^n}{\theta} \equiv 1$, that means that a measurement event is captured at each time step. If some bottlenecks appears in the (queue, processor) pair component, we expect to have a few events travelling through the queue for accurate bottleneck evolution, so that we expect more generally

$$\frac{\tau^n}{\theta} < 1, \tag{17}$$

  but we do not want $\frac{\tau^n}{\theta} << 1$ in order not to handle a large amount of events, what would make loose all the interest of the fluid approach.

- If $\frac{\theta}{\tau_t} \equiv 1$, that means a new event is created and sent each time an event is processed for service time measurement. For same reasons than those exposed above, we ask for

$$\frac{\theta}{\tau_t} < 1, \tag{18}$$

with ratio not too small. For strongly varying flow regimes (nonhomogeneous arrival rate, bottlenecks, etc.), the instantaneous characteristic travel time τ_t can actually vary with a few orders of magnitude. Is this case, the ratio would becomes sometimes very small. We then recommend to also use time-variable sample times θ^n that adapt themselves to local characteristic times of fluid.

- If $\frac{\tau^n}{\tau_t} \equiv 1$, then the numerical scheme propagates the information through the whole computational domain at each time step. Unfortunately, for such hyperbolic transport problems, it is known that explicit approximation schemes need to respect the so-called Courant-Friedrichs-Levy stability criterion ([GR91]) that imposes

  $$\frac{\tau^n}{h} v \;\leq\; \alpha,$$

  for some CFL number $\alpha \leq 1$, $h << 1$ is the characteristic discretization space step. Here $\frac{1}{\tau_t} \equiv v$, that means that the CFL criterion gives

  $$\frac{\tau^n}{\tau^t} \;\equiv\; h.$$

  Because this can become very constraining for time advance, this invites to deals with implicit schemes for unconditional stability. This is the object of the next section. Anyway, in order to be accurate enough with numerical propagation time of order τ_t, we ask for additional condition

  $$\frac{\tau^n}{\tau^t} \; < 1 \tag{19}$$

  with ratio not too small.

Condition (19) is redundant with (17) and (18) but compatible.

5 Numerical approximation and discrete algorithm

Suppose that we use a constant mesh step $h = \frac{1}{M}$ for a given integer M. Let τ^n denote variable time step at time t^n. We define M mesh points $x_j = (j - \frac{1}{2})\,h$, $j = 1..M$ on the interval $[0,1]$. We will often need to use midpoints $x_{j+\frac{1}{2}} = j\,h$, $j = 0, M$. Finally, let us denote $\Phi^n_{i;k} = \Phi_{i;k}(t^n)$, $\Phi^n_i = \Phi_i(t^n)$ and $\Phi^n_o = \Phi_o(t^n)$.

We are going to compute piecewise constant approximations functions $\varphi^{\tau,h}$ with constant values $\rho^n_{k;j}$ on the cell $I_j =]x_{j-\frac{1}{2}}, x_{j+\frac{1}{2}}[\times[t^n, t^{n+1}[$. The variables φ^n_j are also approximations of the value of the exact solutions φ at points (x_j, t^n). The natural and conservative discretization of the initial data in (11) gives

$$\varphi^0_j \;=\; -\frac{1}{2} \quad \text{for } j = 1..M. \tag{20}$$

Suppose that the sequences $(\varphi^n_j)_{j=0,M}$ is known. We want to build a numerical scheme that gives the sequence $(\varphi^{n+1}_j)_j$ at time $t^{n+1} = t^n + \tau^n$. We here suppose that the queue is of infinite capacity. According to (15), (9) and (12) and as suggested in [DV01] for space-time discretization, we then propose:

$$t^{n+1} \;=\; t^n \;+\; \tau^n \tag{21}$$

If $(\Phi^n_i - \Phi^n_o > 0)$, then solve

$$s^{n+1} \;=\; s^n \;+\; \tau^n\,(\Phi^n_i - \Phi^n_o)\,, \tag{22}$$

$$\left(s^{n+1} + \frac{\tau^n}{h}\,(x_{j+\frac{1}{2}}\Phi^n_o + (1 - x_{j+\frac{1}{2}})\Phi^n_i)\right)\,\varphi^{n+1}_j - \frac{\tau^n}{h}(x_{j-\frac{1}{2}}\Phi^n_o + (1 - x_{j-\frac{1}{2}})\Phi^n_i)\,\varphi^{n+1}_{j-1} \;=\; s^n\,\varphi^n_j \;\forall j \tag{23}$$

$$\text{with } \varphi^{n+1}_0 \;=\; \frac{1}{2} + \left[\frac{t^{n+1}}{\theta}\right], \tag{24}$$

else $(\Phi^n_i - \Phi^n_o \leq 0)$ solve

$$s^{n+1} \;=\; s^n \;+\; \tau^n(\Phi^n_i - \frac{s^{n+1}}{s^n}\,\Phi^n_o), \tag{25}$$

$$\left(s^{n+1} + \frac{\tau^n}{h}\,\frac{s^{n+1}}{s^n}x_{j+\frac{1}{2}}\Phi^n_o + \frac{\tau^n}{h}(1 - x_{j+\frac{1}{2}})\Phi^n_i\right)\,\varphi^{n+1}_j - \frac{\tau^n}{h}\left(\frac{s^{n+1}}{s^n}x_{j-\frac{1}{2}}\Phi^n_o + (1 - x_{j-\frac{1}{2}})\Phi^n_i\right)\,\varphi^{n+1}_{j-1} \;=\; s^n\,\varphi^n_j \;\forall j, \tag{26}$$

$$\text{with } \varphi^{n+1}_0 \;=\; \frac{1}{2} + \left[\frac{t^{n+1}}{\theta}\right], \tag{27}$$

where $[x]$ denotes the integer part of x. It is then easy to show both stability result and property:

Proposition 1 *The numerical scheme (20),(21)-(27) is first order accurate in both space and time, is unconditionally L^∞-stable and preserves a local maximum principle for φ.*

We skip the proof because the arguments are similar to those exposed in [DV01]. The maximum principle result states that the discrete solution does not add spurious integer crossings that could be be produced by numerical oscillations. Even if the first order accuracy could seem not to be sufficient, numerical experiments show that it is sufficient when "small" time steps are used.

6 Extension to other models

In the Computational Information Fluid Dynamics context, most of the models of fluid-conserving components lead to conservation equation (2) with a particular propagation velocity v. This a the case for example for the multi-class processor sharing model in [DV01]. Consequently, the conservative equation (3) still holds and the same methodology can be used.

7 Numerical experiments

7.1 Validation on unlimited queue with constant departure rate

Because service times are analytically known in the case of constant departure rates (see (8)), this can help us for implementation validation and numerical error estimates.

Let us consider a FIFO queue fed by two fluid sources. Suppose the initial queue size is 100. The two initial mass fractions are constant equal to 0.5. Suppose a constant outgoing rate $\Phi_o = 1000$ (say packets per second). We choose a piecewise constant time function for $\Phi_i(t)$:

$$\Phi_i(t) \;=\; \max\left(0,\; 1000 + 1000\left(\mathrm{sgn}(\sin(10\pi t)) - 0.4\right)\right).$$

This will lead to a piecewise linear queue size. For space discretization, we choose $h = \frac{1}{M}$ with $M = 80$. For time discretization, we use a variable time step given by

$$\tau^n \;=\; \max\left(\Delta t_{ref},\, 0.005\,\max(1, \frac{s^n}{\Phi_o})\right))$$

where Δt_{ref} is an upper bound of time discretization. The expression ensures both consistency with the equation and compatibility with characteristic times of propagation.

Figure 1 shows the evolution of the computed queue size. In particular, the numerical methods support near zero queue size while preserving positivity. This kind of result is more detailed in [DV01]. On figure 3 are plotted both analytical

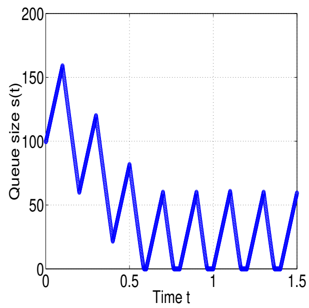

Figure 1: History of FIFO queue size. The numerical scheme preserves the positivity of the queue size.

and numerical computed service times. The numerical results are very satisfactory. The behavior of the response times is qualitatively well reproduced. The committed error is small. Following results deal with the computation of the

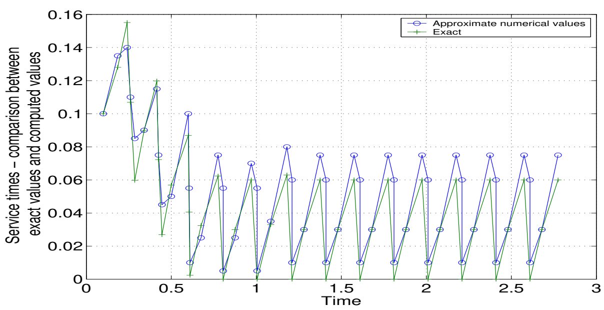

Figure 2: Comparison of services times between exact values and numerical values. The numerical error is quite reasonable and controlled by the discretization parameters.

level function. On figure 3 are represented histories of levels function at three fixed positions, left, middle and right position of the computational domain. The history of the boundary time function $\varphi_{a,\theta}$ is also plotted. What we can see is that the numerical scheme produce an excessive amount of numerical dissipation that makes the level steps be very smeared. Even the step at the first left computational cell are smeared. This is due to the implicit nature of the scheme that introduces nonlocal effects like diffusion. But the excessive of dissipation seems not to be so sensible on performance indicators as commented and observed at figure 3. Finally, figure 4 shows space profiles of the level function at three times

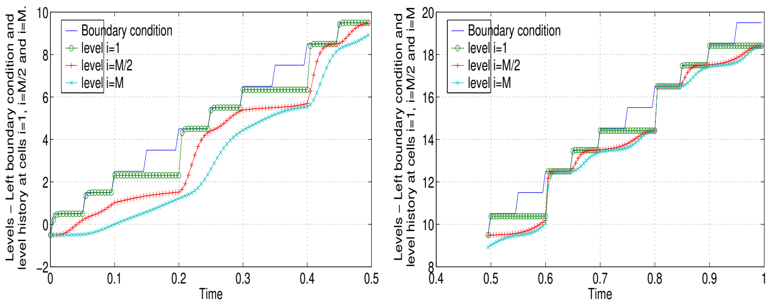

Figure 3: Level history : boundary condition and numerical level history at cells $i = 1$, $i = M/2$ and $i = M$.

$t_1 = 5\,10^{-3}$s, $t_2 = 1.5\,10^{-2}$s and $t_3 = 4.5\,10^{-2}$s. These times are representative to a characteristic time of transport of the first step of the level function that crosses the integer value zero through the computational domain. This again shows the excessive smearing of the steps using an implicit first order accurate scheme. The L^∞-error on level function is large but figure 3 tends to show that the error on service times (a derived information from from level function) is much lower.

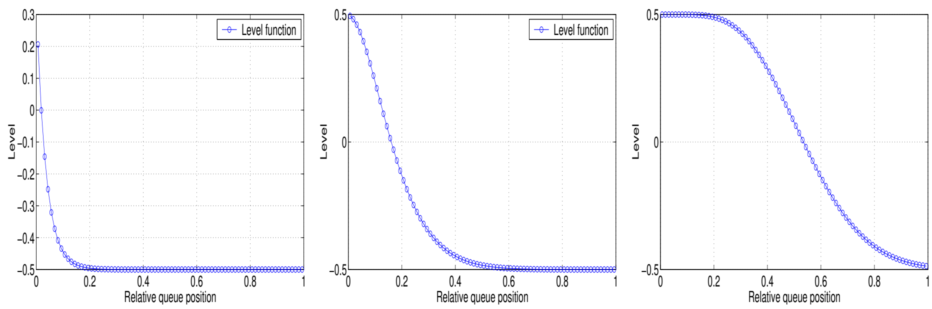

Figure 4: Distribution of the level function at three different times $t_1 = 5\,10^{-3}$s, $t_2 = 1.5\,10^{-2}$s and $t_3 = 4.5\,10^{-2}$s.

7.2 Chaotic departure rate case

A complementary and interesting case is the time-varying departure rate. To show the complete feasibility of the method, we here consider a non smooth rate. This simply models a system with a saturated subsystem downstream the studied one. The downstream component chaotically sends acceptation and refusing messages upstream according to its level of bottleneck. We propose to use as chaotic effective departure rate:

$$\Phi_{eo}(t) \;=\; 1000\ \big(\ 1 - \max\left(0, 0.5\sin(20\pi t) + 0.3\sin(27\pi t) + 0.1\sin(90\pi t)\right)\ \big)\,, \tag{28}$$

where Φ_o plays the role of a crude departure rate. This corresponds to the rate which is limited downstream. Of course, if the queue size reaches zero, we more add a flux limitation reducing this rate. In this test case, initial queue size is chosen in order not to reach zero within the observation time. The downstream limited departure rate is drawn below: Let us more suppose that the queue size

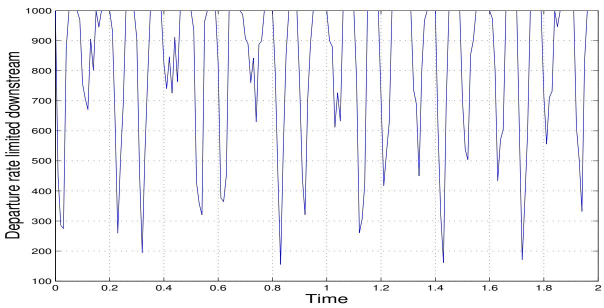

Figure 5: Time departure rate used as exogenous function for this test case.

is not upper bounded, and that the arrival rate is constant equal to

$$\Phi_i \;=\; 820.$$

The initial queue size is set to $s_0 = 50$. We perform a computation between times $t = 0$ and $t = 2s$. The time step is fixed, equal to $10^{-3}s$. The sample rate θ is constant, equal to $5\,10^{-3}s$. A uniform mesh with $M = 80$ computational points is used.

Below is represented a history of the queue size. The time step has been calibrated so that the numerical solution indeed captures the large variations of the continuous solution. Finally, figure 7 shows the history of the level function at

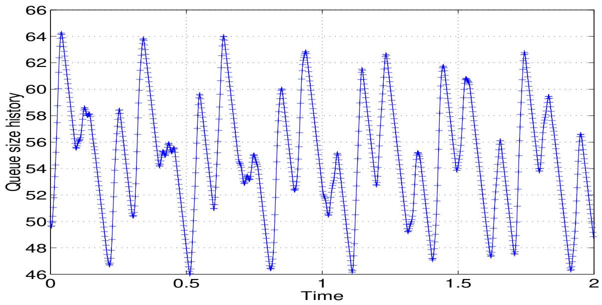

Figure 6: History of the queue size

the last computational cell ($x = 1$), the history of computed service times and the distribution of these times. In particular, the model allows for retrieving the averaged service time and the "variance" of the performance signal.

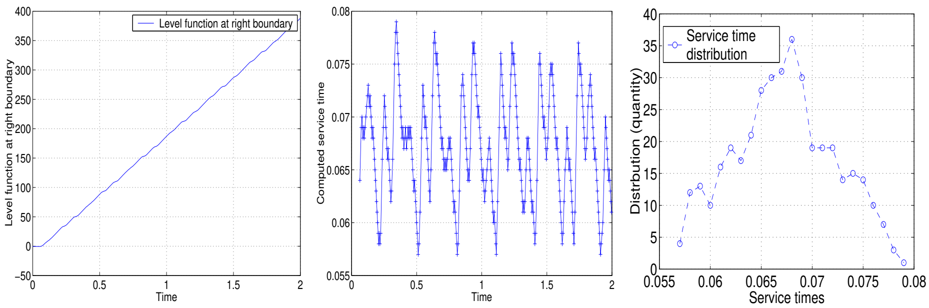

Figure 7: History of the queue size

8 Concluding remarks

We have explored a level set method for service time assessment of buffer systems using fluid PDE transport models. Numerical results are very satisfactory in terms of robustness, numerical complexity and accuracy. Although results have only been performed on single buffer system with FIFO policy, the extension to more complex systems with interacting components should not raise any problem. Because of the ability to use discontinuous level functions, this method can support human interaction and decisions of pointwise delay measurements during run-time for control or supervision. When someone decide to start a new chronometer, then the algorithm makes the level function cross a new integer value for sending an event.
A rigorous work would be to compare performance of event-driven simulation versus this kind of model. This has not been done at the moment. But what we can say for the moment is that the numerical complexity of the method is linear with both network scale and description complexity (the number of fluid to consider for accurate performance assessment). Comparatively, it is known that the numerical complexity of an event-driven modelling technique becomes exponential with network scale and complexity.

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