Source Waves

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Abstract

We propose a new concept of waves for hyperbolic systems with a source term and detail some examples arising in environment science, such as tidal bore waves, surf waves, roll waves, cyclones or hurricanes, and also other applications. These waves behave as solutions of homogeneous linear hyperbolic systems though they satisfy to non linear and non homogeneous systems.

1 Introduction

We propose a new concept of waves as solutions of non linear hyperbolic systems with a source term involving roots and we introduce the following definition:

Definition 1.1 *A Source Wave is a particular solution of a non linear and non homogeneous hyperbolic system whose velocity is entirely determined by a root of the source term.*

Since the root often determines a constant state, the velocity of the wave is a constant in such a case and then corresponds to a linearly degenarated wave. We first propose an example in the scalar case for a one dimension equation. Next we study a set of hyperbolic systems made of two equations in the one dimension case, and give some applications in hydraulics, such as the Roll Waves on a slope, the Tidal Bore Waves in an estuary or the Surf waves on the beach. In the two dimension case we propose a mathematical model which represents some atmospheric phenomena as cyclones or hurricanes. We conclude with other possible applications and remarks.

2 The Scalar case

Let us consider $u = u(x,t)$ with $x \in \mathbb{R}$ and $t > 0$ as the amplitude of a wave whose propagation equation reads

$$u_t \ + \ f(u)_x \ + \ S(u) \ = \ 0 \tag{2.1}$$

with some given regular functions f and S. For more simplicity we suppose f to be a stricly convex function. Now let us imagine that u is also a solution of a linear homogeneous equation of the form

$$u_t \ + \ A \ u_x \ = \ 0 \tag{2.2}$$

with a real A to be well fitted. Some compatibility between (2.1) and (2.2) imposes that A must be some value reached by $f'(u)$ that we denote $A = f'(u_*)$. The difference between (2.2) and (2.1) leads to

$$(A - f'(u)) \ \ u_x \ = \ S(u) \tag{2.3}$$

and since $A = f'(u_*)$ we get $S(u_*) = 0$, that is: u_* must be a root of the source term $S(u)$ at least for $u_x \neq 0$ which is the case we are concerned with, a "non flat" wave. Reversely if u_* is a root of $S(u)$ a solution of (2.2) may be also a solution of (2.1) for $A = f'(u_*)$ and reads $u(x,t) = u_0(x - f'(u_*)t)$ where u_0 stands for the Cauchy data for u. We write

$$S(u) = -R_*(u)\,(f'(u) - f'(u_*)) \tag{2.4}$$

since u_* is a root of $S(u)$, and suppose $R_*(u) > 0$ and regular enough. Then (2.3) reads

$$(f'(u_*) - f'(u))\,(u_x - R_*(u)) \;=\; 0$$

that is

$$u_x \;=\; R_*(u). \tag{2.5}$$

Now we introduce a function $\psi_*(u)$ as a primitive of $\frac{1}{R_*(u)}$, thus $\psi_*'(u) \;=\; \frac{1}{R_*(u)}$. From (2.5) we have $\psi_*'(u)u_x \;=\; 1$ which gives after integration with respect to x,

$$\psi_*(u) \;=\; x \;-\; K(t) \tag{2.6}$$

with some function $K(t)$ not dependent on x. By differentiating with respect to t we have

$$K'(t) = -\frac{u_t}{R_*(u)} = f'(u)\frac{u_x}{R_*(u)} \;-f'(u) + f'(u_*) = f'(u_*)$$

since $\frac{u_x}{R_*(u)} = 1$. We get $K(t) = f'(u_*)t - K_0$ for some constant K_0 and now (2.6) becomes

$$\psi_*(u) \;=\; x \;-f'(u_*)t \;+\; K_0$$

which says that $\psi_*(u)$ and then also u is a solution of the linear homogeneous equation (2.2) with $A = f'(u_*)$.The function ψ_*^{-1} will be called the *profile function* since

$$u(x,t) \;=\; \psi_*^{-1}\,(x - f'(u_*)t + K_0) \tag{2.7}$$

gives the profile of the wave represented by the solution. We have $\dfrac{d\;\psi_*^{-1}}{d\xi}(\psi_*(u)) = R_*(u) > 0$ and then u is an increasing function. This is compatible with the assumption of strictly hyperbolocity of f together with the positivity of R_*.

Since a bounded solution is expected, u will decrease somewhere, which can be done only through a shock wave. The Rankine Hugoniot condition applied to (2.1) imposes that the shock propagates with the velocity

$$x'(t) = \frac{f(u_2) - f(u_1)}{u_2 - u_1} \tag{2.8}$$

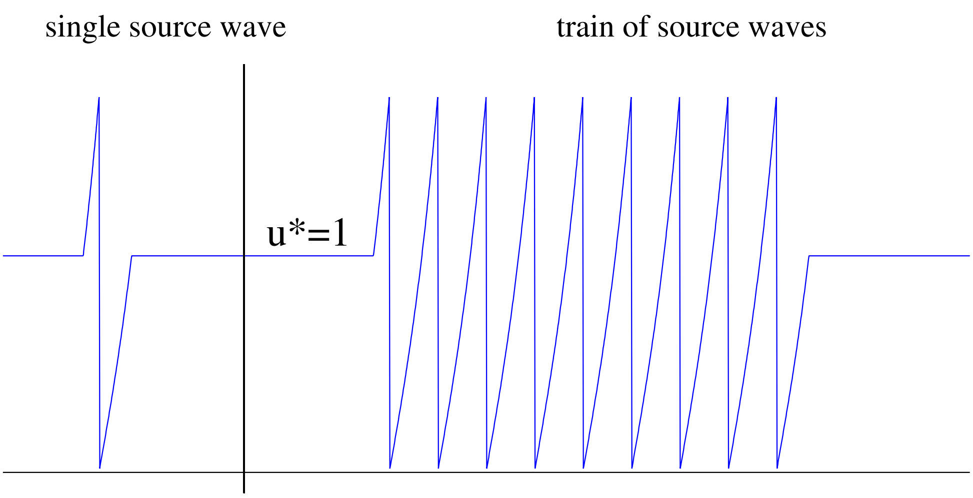
where u_1 and u_2 denote the values of the solution on both sides of the discontinuity, with u_1 on the left and u_2 on the right. Since f is a convex function, the entropy condition imposes that $u_1 > u_2$. From (2.2) with $A = f'(u_*)$, this velocity must be also equal to $f'(u_*)$. We get the condition

$$f'(u_*) \;=\; \frac{f(u_2) - f(u_1)}{u_2 - u_1}\;. \tag{2.9}$$

This gives the shape of a single source wave: the amplitude starts from the value u_* and increase towards some value u_1 with a profile imposed by (2.7), then we have the shock discontinuity and the wave restarts with a value u_2 linked to u_1 and u_* by the condition (2.9) and increase to reach the value u_* still following the profile (2.7). Since f' is an increasing function, the condition (2.9) tells us that $u_2 < u_* < u_1$, which is compatible with the previous shape description. We propose a non trivial example computed with $f(u) = \frac{u^3}{3}$ and $u_* = 1,\ u_1 = 0$. This imposes $u_2 = \sqrt{3}$, and $R_*(u) = \dfrac{1+u+u^2}{3\,(1+u)}$ and then the profile function is the inverse of

$$\psi(u) = \frac{3}{2}\ ln\left(1+u+u^2\right) + \sqrt{3}\left[Atan\left(\frac{1+2u}{\sqrt{3}}\right) - \frac{\pi}{6}\right].$$

A train of source waves is obtained by concatenating a sequence of single source waves. We may remark that we can concatenate different source waves, that is with different values of $u_1 > u_*$ but with the same u_*. The value of u_2 is deduced from u_1 and u_* by solving (2.9) with $u_2 < u_* < u_1$



example with f(u) = u^3/3

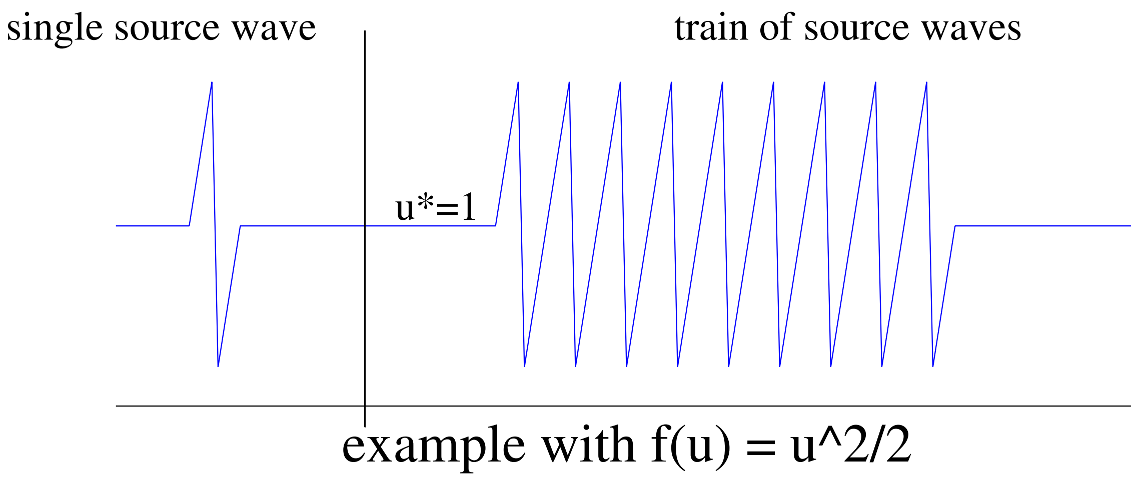
In the case of the Burger equation, that is for (2.1) with $f(u) = \frac{u^2}{2}$, and $S(u) = -k\,(u-u_*)$ for some constant $k \geq 0$, we have $R_*(u) = k$ and $\psi_*'(u) = \frac{1}{k}$ then $\psi_*(u) = \frac{u}{k} = x - u_*t + x_0$ which gives $u = k\,(x - u_*t + x_0)$. For any $R > 0$ we introduce the function χ_R defined by

$$\chi_R(\xi) \;=\; \begin{cases} \frac{1}{2}\left(|\xi| - R\right)^2 & for \;\; |\xi| < R \;, \\ 0 & for \;\; |\xi| \geq R \;. \end{cases} \tag{2.10}$$

then a single source wave has the form

$$u(x,t) \;=\; u_* \;+\; k\, \frac{\partial}{\partial x}\chi_R\left(x - u_* t + x_0\right) \;\;. \tag{2.11}$$

We notice that u is equal to u_* but on an interval of length $2R$ which is moving with the velocity u_*, and a shock occurs in the middle of this interval. A train of such source waves may be constructed by concatenating several single source waves.



This source wave can be generalized in 2, 3 or more dimensions since

$$U \equiv \begin{pmatrix} u \\ v \\ w \end{pmatrix} \;=\; U_* + k\, \nabla \chi_R \left(\sqrt{|x - u_* t|^2 + |y - v_* t|^2 + |z - w_* t|^2} \right) \quad with \;\; U_* = \begin{pmatrix} u_* \\ v_* \\ w_* \end{pmatrix} \tag{2.12}$$

is a source wave solution to

$$\frac{\partial}{\partial t} U \; + \; (U.\nabla)\, U \; = \; k\,(U - U_*) \tag{2.13}$$

which can modelize the velocity of some kind of "black hole".

3 Source Waves in Hydraulics

3.1 A derivation of the Saint-Venant model

We first propose a construction of the Saint-Venant model which does not reduce to some averaging and approximation of the Navier-Stocke equations. We assume some minimal hypotheses which are sufficient for the derivation of the model. For simplicity the bottom is supposed to be flat and we write the model in the one dimension case. The generalizations to a variable bottom or the two dimension case are obvious and studied later.

First we claim that wa are concerned with the depth of water $q > 0$ which is transported by a flux m, and that the mass of water is conserved. This corresponds to the conservation law

$$q_t \; + \; m_x \; = \; 0 \; . \tag{3.1}$$

Next we propose the following assumptions:

$(H0)$ At most two waves of different velocities are expected and these velocities are regular functions of q and m;

$(H1)$ There is no flux on a dry bottom;

$(H2)$ The flux m is a conservative quantity;

$(H3)$ The variation of the velocity does not depend on the depth of water.

From $(H0)$ we know that the model is made of two equations corresponding to a strictly hyperbolic system which reads

$$m_t \; + \; (\lambda_1 + \lambda_2) \; m_x \; - \lambda_1 \lambda_2 \; q_x \; = \; 0 \quad , \tag{3.2}$$

where the two velocities are denoted λ_1 and λ_2, with for instance $\lambda_1 < \lambda_2$. Both are function of q and m. By performing a Galilean transform, that is changing x into $x + \alpha t$ with some constant α and the same t, it appears that λ_1 and λ_2 are changed respectively into $\lambda_1 + \alpha$ and $\lambda_2 + \alpha$, and that the flux m is changed into $m + \alpha q$ but q is not changed. We define $u = \frac{\lambda_1 + \lambda_2}{2}$ and $c = \frac{\lambda_2 - \lambda_1}{2}$. It appears that u is changes into $u + \alpha$ but c is not changed. From these observations we can deduce that u depends on q and m, that is $u = u(q, m)$, but c depends only on q, that is $c = c(q)$ or $\frac{\partial c}{\partial m} \; = \; 0$. We also notice that the new form of (3.2) using u and c reads

$$m_t \; + \; 2u \; m_x \; + \; \left(c^2 - u^2\right) \; q_x \; = \; 0 \; . \tag{3.3}$$

From $(H2)$ we know that (3.3) can be written under the form

$$m_t \; + \; \left(F(q,m)\right)_x \; = \; 0 \tag{3.4}$$

with F satisfying $\dfrac{\partial F}{\partial m} = 2u$, $\dfrac{\partial F}{\partial q} = c^2 - u^2$. By computing in two different ways $\frac{\partial^2 F}{\partial q \partial m}$ and using $\frac{\partial c}{\partial m} = 0$ we get that u must satisfy the scalar equation

$$\frac{\partial u}{\partial q} + u \frac{\partial u}{\partial m} = 0 , \tag{3.5}$$

that is the Burger equation using the variables q and m. The characteristic theory lells us that a regular (from($H0$)) solution has the form $u = H(m - qu)$ where H must be an increasing function from the regularity hypothesis (else a shock will occur). By inverting this equation we get $m = q\,u + H^{-1}(u)$, that is a straight line for $u = \textit{constant}$ in the phase plane (q, m). From ($H1$) this line must pass throught the state $(0, 0)$, which gives $H^{-1}(u) = 0$, thus

$$u = \frac{m}{q} . \tag{3.6}$$

We get that u corresponds to the usual velocity, and that no averaging nor approximation were needed to obtain the result, as in the usual construction of the Saint Venant model.

We compute now the acceleration $u_t + uu_x$ and obtain easily from (3.1), (3.3) and (3.6) the equation

$$u_t + u\,u_x + \frac{c(q)^2}{q}\,q_x = 0 \tag{3.7}$$

and use ($H3$) to claim that the term $\dfrac{c(q)^2}{q}$ must be a constant, which is usually denoted g as the gravity constant. Then $c = \sqrt{gq}$ is called the critical speed, corresponding to the soundspeed in hydrodynamics, and the number $F = \frac{|u|}{c}$ is called the Froude number, corresponding to the Mach number in hydrodynamics too.

We have derived that way the Saint-Venant model made of (3.1) and (3.3) with $u = \frac{m}{q}$ and $c = \sqrt{gq}$. In the general case when a topography $z = a(x)$ and a friction term are given, (3.3) must be replaced by

$$m_t + 2u\,m_x + \left(c^2 - u^2\right)\,q_x + c^2 a'(x) + k\,|u|\,u = 0 \tag{3.8}$$

in the case of the Strickler friction term of coefficient $k > 0$ and with the same u and c. We may remark that the assumption ($H3$) corresponds to some "shallow water" hypothesis, more exactly we only suppose the water to be not too deep. As a matter of fact, for a depth of 100 meters for instance, a wave will travel on about 10 meters during a timestep of one second. During the same time at least 8 interactions with the bottom will occur since the sound speed in the water is close to 1600 meters pro second. This is enough to stabilize the flow and ensure that the Saint-Venant model is suitable up to such a depth. For a depth of 1000 meters, 1.25 second is needed before the first interaction takes place, and during that time the wave has travelled on 124 meters, which is too much and makes the model of Saint-Venant unsuitable in this case.

3.2 The Saint-Venant source waves

We consider for simplicity the case where $a'(x)$ is a constant, that will be denoted p and corresponds to a constant slope of the bottom. We define $\lambda = \sqrt{\dfrac{|p|}{k}}$ and $s_p = sign(p)$, and then we set

$$S(q, m(q)) = \frac{k}{q^2} \left(g \; s_p \; \lambda^2 \; q^3 + |m(q)| \; m(q) \right) \tag{3.9}$$

as a new notation of the source term.

We are looking for a solution of (3.1), (3.8) corresponding to a source wave, that is travelling with a constant velocity A. If we describe the wave using the parameter q we expect A such that

$$q_t + A \, q_x = 0$$

and if q is related to m by $m = m(q)$, we get

$$m'(q) = A$$

as a necessary condition from (3.1). Next we get by integrating (3.2)

$$A \, q = m(q) + B \tag{3.10}$$

with some constant B to be fitted later. We have the following expression for the velocity

$$u = \frac{m(q)}{q} = A - \frac{B}{q} \, . \tag{3.11}$$

We put this expression into (3.8) and obtain, using (3.2), the equation

$$\left[\, (A-u)^2 - c^2 \right] \, q_x = S(q, m(q)) \, ,$$

or

$$\frac{\left((A-u)^2 - c^2 \right)}{k \; (g \; s_p \lambda^2 \; q^3 + |m(q)| \, m(q))} \, q_x = 1 \, , \tag{3.12}$$

which also reads

$$\left[B^2 - c^2 \, q^2 \right] \, q_x = q^2 \; S(q, m(q))$$

that is

$$\frac{B^2 - c^2 q^2}{k \; (g \; s_p \; \lambda^2 \; q^3 + |m(q)| \; m(q))} \, q_x = 1 \, . \tag{3.13}$$

This formula is the analogue of (2.6) in the scalar case with $\psi_*(q)$ such that

$$\psi'_*(q) = \frac{B^2 - c^2 \; q^2}{k \; (g \; s_p \; \lambda^2 q^3 + |m(q)| \; m(q))} \tag{3.14}$$

Now (3.13) reduces to $\psi_*'(q)\ q_x\ = 1$ and we get

$$\psi_*(q) \ = \ x \ - \ At \ + \ x_0 \tag{3.15}$$

after integration and identification.

The relation (3.13) tells us that there is a critical water depth q_* for which $B^2 \ = \ c(q_*)^2 q_*^2$, that is

$$q_* \ = \ \left(\frac{B^2}{g}\right)^{\frac{1}{3}} . \tag{3.16}$$

In this case, (3.13) imposes that q_* must be also a root of the denominator (otherwise the back of the wave is not bounded). We get that $u_* = A - \frac{B}{q_*} \ < \ 0$ for $s_p = 1$, $u_* > 0$ for $s_p = -1$, that is $s_p u_* < 0$ or $u_*^2 = -s_p\,|u_*|\,u_*$. We get that way $g\ \lambda^2\ q_*^3 \ = \ (Aq_* - B)^2$, and since $gq_* = c(q_*)^2$, we get the relation $|Aq_* - B| \ = \ \lambda\ q_*\ c(q_*)$ and dividing by q_*,

$$|u_*| \ = \ \lambda\ c(q_*)$$

which leads to the following result:

Proposition 3.1 *The number $\lambda = \sqrt{\frac{|p|}{k}}$ is equal to the Froude number for the critical value q_*, that is $\lambda \ = \ \dfrac{|u_*|}{c(q_*)}$.*

Let us consider now a shock wave and denote by q_s, $m_s = u_s q_s$ the state on the left side of the discontinuity and by q_i, $m_i = u_i q_i$ the state on the right side. We suppose $B > 0$. In this case the Entropy condition imposes that $q_s > q_i$, which means that a shock profile is a decreasing one. It is the reverse for $B < 0$. From the Rankine Hugoniot condition the shock velocity is given by

$$x'(t) \ = \ \frac{u_i + u_s}{2} \ + \ \frac{q_i + q_s}{2}\ \sqrt{g\ \frac{q_i + q_s}{2q_i q_s}} \tag{3.17}$$

Since we are concerned with a source wave, we have

$$u_i \ = \ A \ - \ \frac{B}{q_i} \quad , \quad u_s \ = \ A \ - \ \frac{B}{q_s} \quad , \quad A \ = \ x'(t)\ ,$$

with the same A and B, which transform (3.17) into

$$A \ = \ A \ - \ B\ \frac{q_i + q_s}{2q_i q_s} \ + \ \frac{q_i + q_s}{2}\ \sqrt{g\frac{q_i + q_s}{2q_i q_s}}$$

and we get

$$B \ = \ \sqrt{g\ \frac{q_i + q_s}{2}\ q_i\ q_s} \tag{3.18}$$

We come back now to the critical value q_* given by (3.16) and we get the following result.

Proposition 3.2 *The critical depth q_* is an average value between q_s and q_i given by*

$$q_* \;=\; \left[\, \frac{q_i + q_s}{2}\; q_i\; q_s \,\right]^{\frac{1}{3}} . \tag{3.19}$$

This means that the critical value q_* given in (3.16) is always one of the possible water depth, corresponding to a Froude number equal to λ and also to a root of the source term, which characterizes a source wave. We have the general following form for A and B

$$A \;=\; \pm\;(\;1\;\pm\;\lambda)\;c_* \quad , \quad B^2 \;=\; g\;q_*^2 \;=\; c_*^2\;q_*^2 \tag{3.20}$$

where the signs of A and B depend on the application.

3.3 The Roll waves

The Roll wave phenomenon is described and modelized for instance by R.F.Dressler in [1] or J.J.Stokker in [2]. These waves appear in a flow moving downwards in a channel. These Roll waves actually corresponds to Source waves for the SaintVenant equation. The next figure below gives the shape of an example of Roll wave computed as a Source wave, which is detailed later.

We have $s_p = -1$ and $m(q) > 0$. The critical depth q_* corresponds to the mean depth of the water flow (as in (3.19)) and we have $A = (1+\lambda)\;\; c_*$ and $B = c_* q_*$. Hence (3.14) becomes

$$\psi_*'(q) \;=\; \frac{c_*^2\; q_*^2 \;-\; c^2\; q^2}{k\;\left(\left((1+\lambda)\,q - q_*\right)^2 c_*^2 \;-\; g\lambda^2 q^3\right)}$$

We set $\xi \;=\; \frac{q}{q_*}$ whose critical value is 1 and we get

$$\psi_*'(q) \;=\; \psi_*'(\xi q_*) \;=\frac{1}{k}\;\frac{1\;-\;\xi^3}{\left((1+\lambda)\,\xi\;-\;1\right)^2\;-\;\lambda^2\;\xi^3} \;=\frac{1}{k}\;\frac{1\;+\;\xi\;+\;\xi^2}{\lambda^2\xi^2\;-\;(1+2\lambda)\,\xi\;+\;1} \tag{3.21}$$

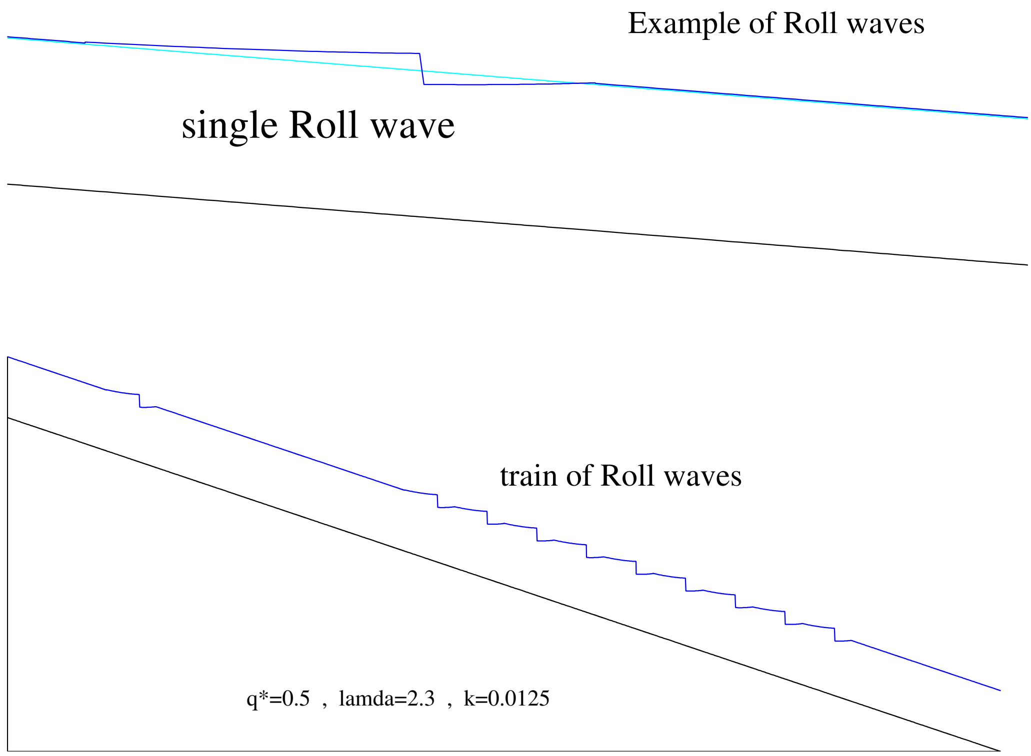
where the denominator has the following roots

$$\xi_1 \;=\; \frac{1+2\lambda\;-\;\sqrt{1+4\lambda}}{2\;\lambda^2} \quad , \quad \xi_2 \;=\; \frac{1+2\lambda\;+\;\sqrt{1+4\lambda}}{2\;\lambda^2} \tag{3.22}$$

We note that $0\; <\; \xi_1\; < min\,(\xi_2, 1)$, ξ_1 goes to 1 and ξ_2 goes to infinity as λ goes to zero. By integrating (3.21) we get explicitly the function ψ_*, which reads

$$\psi_*(q) \;=\; \frac{q}{k\;\lambda^2} \;+\; \frac{q_*}{k}\;\left[\;C\;ln\left|\frac{q-q_1}{q_*}\right| \;+\; D\;ln\left|\frac{q-q_2}{q_*}\right|\right] \;+\; Constant \tag{3.23}$$

where $q_1 \;=\; \xi_1\; q_*$, $q_2 \;=\; \xi_2\; q_*$ and



$$C \;=\; \frac{1}{2\lambda^4\sqrt{1+4\lambda}}\;\left(\left(\lambda+1\right)^2\sqrt{1+4\lambda}\;-\;\left(1+4\lambda+3\lambda^2+2\lambda^3+2\lambda^4\right)\right)$$

$$D \;=\; \frac{1}{2\lambda^4\sqrt{1+4\lambda}}\;\left(\left(\lambda+1\right)^2\sqrt{1+4\lambda}\;+\;\left(1+4\lambda+3\lambda^2+2\lambda^3+2\lambda^4\right)\right)$$

The function ψ_* is defined as a continuous function for $q > q_2$. Since the effective values of q stay between the two jump values q_i and q_s we need $q_2 < q_i < q_*$, which gives the necessary condition $q_2 < q_*$ or using the notation $\xi = \frac{q}{q_*}$, $\xi_2 < 1$. This requires $\sqrt{1+4\lambda} < 2\lambda^2 - 2\lambda - 1$ or $4\lambda^3\left(\lambda-2\right) > 0$, which leads to the following result.

Proposition 3.3 *If $\lambda < 2$, a Roll wave cannot appear.*

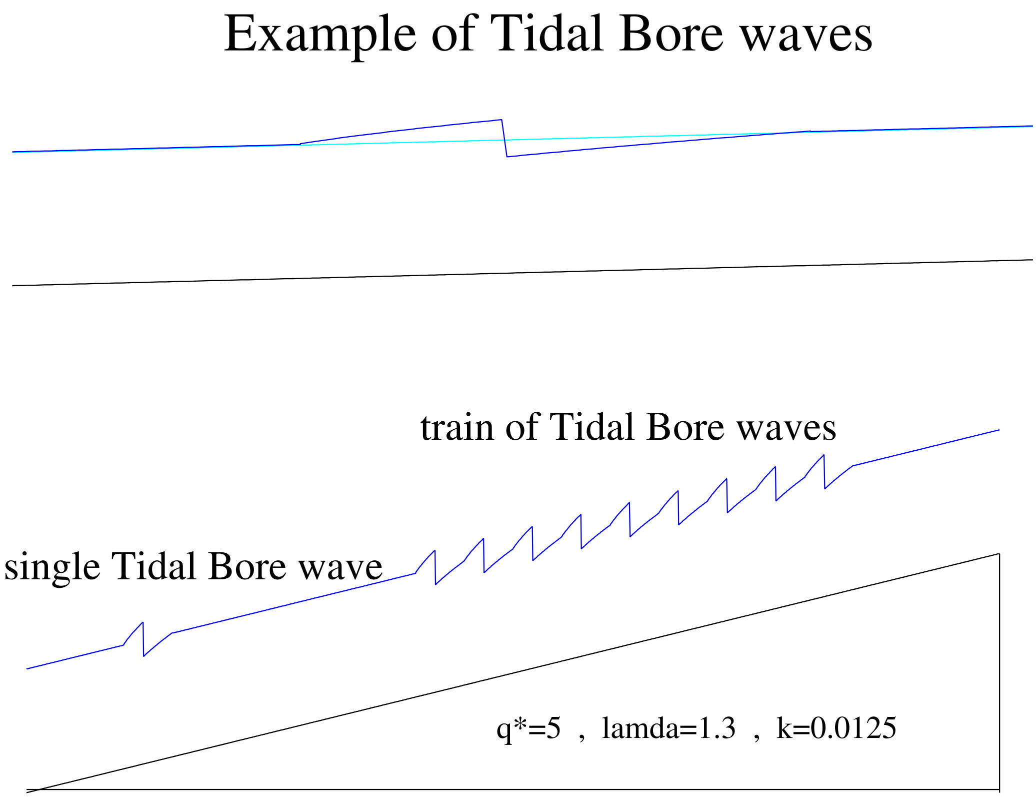
The figure above shows an example of Roll waves. We have taken $q_i = \alpha q_* + (1-\alpha)\, q_2$ with $\alpha = 0.4$ and q_s is obtained from (3.19) for $q_s > q_i$, that is $q_s = \xi_s q_*$ with

$$\xi_s \;=\; \frac{\xi_i}{2}\;\left(\sqrt{1+\frac{8}{\xi_i^3}}\;-\;1\right)\quad,\quad \xi_i \;=\; \alpha\;+\;\left(1-\alpha\right)\;\xi_2\;.$$

The wavelength is deduced immediately from (3.15) and is equal to $\psi_*(q_s) - \psi_*(q_i)$.
The picture is obtained by using (3.15) with the explicit value of ψ_* computed above in (3.23).
The first one zooms one wave, and the second one is a concatenation of several waves.

4 The Tidal Bore waves

A tidal bore is a wave or a series of waves propagating upstream as the tidal flow turn to rising. A detailed description and analysis is to be found for example in [7] or [8] for instance. We use the Saint Venant equations with the river coming downstream from the right, that is with a negative velocity ($m(q) < 0$) and an increasing slope ($s_p = 1$). The critical depth q_* corresponds to a mean depth for which the phenomenon is possible. We have, as for the general case a wave velocity equal to $A = (1+\lambda)\ c_*$ with $c_* = \sqrt{gq_*}$ and a velocity $u = A - \frac{B}{q}$ with $B = c_* q_*$.



We use again the notation $\xi = \frac{q}{q_*}$ whose critical value is 1 and we get from (3.14)

$$\psi_*'(q) \ = \ \psi_*'(\xi q_*) \ = \ \frac{1}{k}\,\frac{(1-\xi^3)\ c_*^2\ q_*^2}{g\lambda^2\xi^3 q_*^3 - \left((1+\lambda)\,c_*\xi q_* \ - \ c_* q_*\right)^2}\ ,$$

which reduces to

$$\psi_*'(q) \;=\; \frac{1}{k}\;\frac{1 \;+\; \xi \;+\; \xi^2}{-\lambda^2\xi^2 \;+\; (1+2\lambda)\,\xi \;-\; 1} \tag{4.1}$$

The roots of the denominator are the same as in (3.22) before, that is

$$\xi_1 \;=\; \frac{1+2\lambda \;-\; \sqrt{1+4\lambda}}{2\;\lambda^2} \quad , \quad \xi_2 \;=\; \frac{1+2\lambda \;+\; \sqrt{1+4\lambda}}{2\;\lambda^2}$$

and since only positive values of $\psi_*'(q)$ are expected, the only allowed values for ξ lay between these roots. We always have $0 < \xi_1 < \xi_2$ and $\xi_1 < 1$. We have $\xi_2 > 1$ only for $\lambda < 2$, which leads to the following result

Proposition 4.1 *A Tidal Bore wave is expected only for a Froude number $\lambda < 2$.*

By integrating (4.1) we get the inverse of the profile function

$$\psi_*(q) \;=\; -\,\frac{q}{k\lambda^2} \;+\; \frac{q_*}{k}\,\left[C\;ln\left(\frac{q-q_1}{q_*}\right) \;-\; D\;ln\left(\frac{q_2-q}{q_*}\right)\right] \;+\; Constant \tag{4.2}$$

with $q_1 = \xi_1 q_*$, $q_2 = \xi_2 q_*$ and

$$C \;=\; \frac{1}{2\;\lambda^4\,\sqrt{1+4\lambda}}\;\left(\,1+4\lambda+3\lambda^2+2\lambda^3+2\lambda^4 \;-\; (1+\lambda)^2\;\sqrt{1+4\lambda}\right)\;,$$

$$D \;=\; \frac{1}{2\;\lambda^4\,\sqrt{1+4\lambda}}\;\left(\,1+4\lambda+3\lambda^2+2\lambda^3+2\lambda^4 \;+\; (1+\lambda)^2\;\sqrt{1+4\lambda}\right)\;.$$

The function ψ_* is defined as a continuous function for $q_1 < q < q_2$ with $q_2 > q_*$ from the hypothesis $\lambda < 2$.

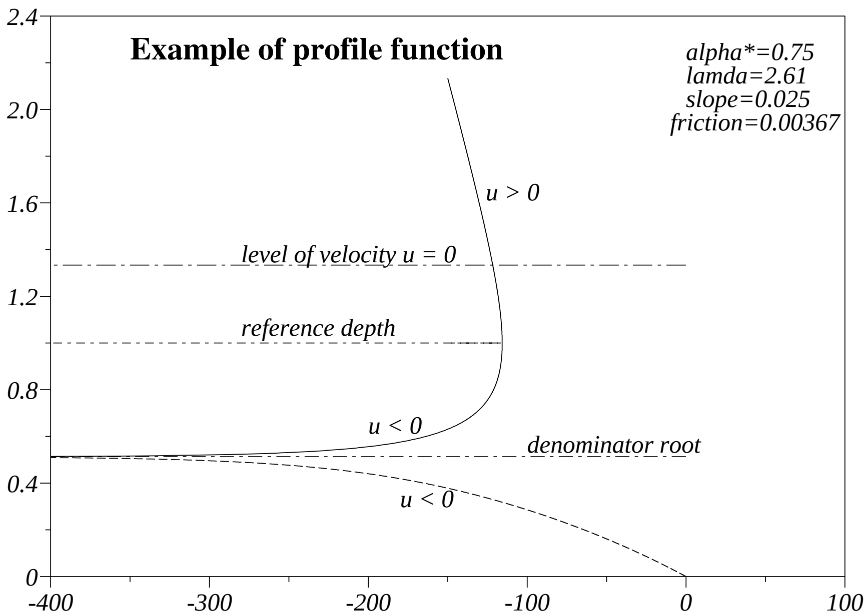
The figure above shows an example of Tidal Bore wave. We have taken $q_i = \alpha q_* + (1-\alpha)q_1$ with $\alpha = 0.8$ and computed q_s as before for the Roll wave, that is $q_s = \xi_s q_*$ with

$$\xi_s \;=\; \frac{\xi_i}{2}\,\left(\sqrt{1+\frac{8}{\xi_i^3}} \;-\; 1\right) \quad , \quad \xi_i \;=\; \alpha \;+\; (1-\alpha)\;\xi_1\;.$$

As before, the wavelength is deduced immediately from (3.15) and is equal to $\psi_*(q_s) - \psi_*(q_i)$. The picture is obtained by using (3.15) with the explicit value of ψ_* computed above in (4.2). The first one zooms one wave, and the second one is a concatenation of several waves.

5 The waves in the Surf Zone

We consider here the propagation of the water waves in the surf zone, on a beach with a constant slope. These waves are already shaped from the swell and will become breakers by leaving the surf zone. A classification of waves and many other details on this topics may be found in [9]. We suppose that the bottom of the beach is increasing from the left to the right, which means that the shore is on the right side and the open sea on the left side, and the slope is positive ($s_p = 1$). We use the same SaintVenant model as above, but this case is a little different since the sign of the velocity u is not constant. The velocity u is negative in the lower part of the wave and positive in the higher part. Hence, for a given wave, the sign of the velocity u will change once on the back of the wave, and a second time in the shock at the wave front. Another difference is the fact that the value of the reference depth q_* is not an allowed value for the wave. The physical meaning of q_* will be detailed farther.



The inverse profile function reads

$$\psi'_*(q) = \frac{1-\xi^3}{k\ \left(\lambda^2 \xi^3\ +\ |\alpha_* \xi - 1|\ (\alpha_* \xi - 1)\right)} \tag{5.1}$$

with $\xi = \frac{q}{q_*}$, $B = c_* q_*$ and $A = \alpha_* c_*$ with some constant α_* to be fitted later. The possible profiles are presented on the picture above.
We notice that the back of the wave will always have a negative velocity since an increasing profile is required. The denominator in (5.1) always has a root corresponding to a negative velocity ($u < 0$). The Rankine Hugoniot condition imposes a shock velocity which is different from $A = \alpha_* c_*$, so that this cannot correspond to a source wave. However the part of the profile situated below the level corresponding to the denominator root is suitable to represent the last wave close to the shore, flowing back to the sea. This remark is valid for any positive constant value of α_*.

The preceeding argument tells us that a variable wave velocity A must be taken in account. We prepare to use a velocity depending on the variable x, that is the distance to the shore, for the source wave.We introduce the level difference z between the effective depth q and a reference depth q_* which depends on the space variable x, that is

$$z\ =\ q\ -\ q_*(x)\ .$$

We have $z_t = q_t$ which leads to the equation

$$z_t\ +\ m_x\ =\ 0\ .$$

We propose to choose q_* as the profile at rest, that is

$$q_*(x)\ =\ p\ (L - x) \tag{5.2}$$

where p is the slope and L is such that the position of the shore at rest corresponds to $x = L$. We get

$$z_x\ =\ q_x\ +\ p,$$

and a new system made of three equations

$$\begin{array}{cccc} z_t & + & m_x & = & 0 \\ m_t & + & 2um_x - u^2 q_x + c^2 z_x & = & -k\,|u|\,u \\ q_t & + & uq_x + m_x - uz_x & = & -pu \end{array} \tag{5.3}$$

This is an hyperbolic system whose eigen velocities are $u - c$, u and $u + c$ and the root of the source term corresponds to $u_* = 0$. This system is not written under a conservative form, and we shall use the Rankine Hugoniot condition derived from the previous form for q and m to manage the shock waves. The main idea we introduce here is to change the roles of q and z for the description of the source wave. The main linkage appears now between m and z instead of m and q. We look for a formula of the following form

$$m \;=\; A(x)\; z \;-\; B(x) \tag{5.4}$$

The root of the source term, which is $u_* = 0$, describes the shape of the water surface at rest, which corresponds to $q = q_*(x)$ from our previous choice of q_*, hence to $z = 0$, which implies that $B(x)$ must be zero, that is $B(x) = 0$. The linear equation for the source wave becomes

$$z_t \;+\; \left(A(x)z\right)_x \;=\; 0 \tag{5.5}$$

and we have

$$m_t \;=\; A(x)\; z_t \;=\; -A(x)A'(x)\; z \;-\; A(x)^2\; z_x$$

and

$$q_x \;=\; z_x \;-\; p\;.$$

The second equation in (5.3) becomes

$$\left(A(x)^2 \;-\; 2uA(x)\; + u^2 \;-\; c^2\right)\; z_x \;+\; \left(A(x)A'(x) - 2uA'(x)\right)\; z \;=\; p\; u^2 \;+\; k\; |u|\,u,$$

that is

$$\frac{\left((A(x)-u)^2 \;-\; c^2\right)\; z_x \;+\; \left(A(x)-2u\right) A'(x)\; z}{p\; u^2 \;+\; k\; |u|\,u} \;=\; 1 \tag{5.6}$$

which is a generalization of (3.12) when A depends on x and with u instead of λ.
We consider now a shock wave of velocity $A(x)$. The Rankine Hugoniot condition is the same as before for the $q-m$ formulation and since the jumps for z and q are the same, we get by denoting $q_1 = q_1(x)$ and $q_2 = q_2(x)$ the values of q on both sides of the discontinuity,

$$A(x) \;=\; A(x) \;+\; A(x)\; p\; (x-L)\; \frac{q_1+q_2}{2q_1q_2} \;+\; \frac{q_1+q_2}{2}\; \sqrt{g\; \frac{q_1+q_2}{2q_1q_2}}\;,$$

which reduces to

$$A(x)\; p\; (L-x) \;=\; \sqrt{g\; q_1\; q_2\; \frac{q_1+q_2}{2}} \tag{5.7}$$

Since we have chozen $q_*(x) \;=\; p\; (L-x)$, we also expect to have

$$q_*(x)^3 \;=\; q_1\; q_2\; \frac{q_1+q_2}{2} \tag{5.8}$$

as before. This implies

$$A(x)\ q_*(x) \ = \ \sqrt{g}\ q_*(x)^{\frac{3}{2}}$$

and leads to the following expression of $A(x)$,

$$A(x) \ = \ \sqrt{g\ q_*(x)} \ = \ \sqrt{g\ p\ \ (L-x)}\ . \tag{5.9}$$

Now we can rewrite (5.6) using $u \ = \ A(x)\ \frac{z}{q}$ and get

$$\frac{\left(A(x)^2\left(1-\frac{z}{q}\right)^2 \ - \ c^2\right)\ z_x \ + \ A(x)A'(x)\left(1-2\frac{z}{q}\right)\ z}{\frac{A(x)^2}{q^2}\ z\ \left(p\ z \ + \ k\ \ |z|\right)} \ = \ 1\ ,$$

or under the following differentiel equation forms

$$z_x \ = \ \frac{\frac{p}{2}\ \left(A(x)^2 \ - \ g\ z\right)^2 \ - \ k\ g\ A(x)^2\ |z|}{3\ A(x)^4 \ + \ 3\ A(x)^2\ g\ z \ + \ g^2\ z^2}$$

or

$$z_x \ = \ \frac{p}{6}\ \frac{\left(p\ \ (L-x) \ - \ z\right)^2 \ - \ 2\ k\ \ (L-x)\,|z|}{p^2\ \ (L-x)^2 \ + \ p\ \ (L-x)\ \ z \ + \ \frac{z^2}{3}}\ . \tag{5.10}$$

We notice that for $z=0$ the slope of the wave is $z_x \ = \ \frac{p}{6}$ and that this slope vanishes twice for $z>0$ when the friction coefficient k is not zero. One of these values can be reached in practice, and reads

$$z \ = \ (L-x)\ \left(p \ + \ 2k \ - \ 2\ \sqrt{k\,(k+p)}\right)\ ,$$

which is always between 0 and $p\,(L-x)$.

6 Construction of the wave train in the surf zone

We need to know the period T of the waves, which is a constant and depends on the swell from the open sea. We define a single wave as the part of the shape between two consecutive zero level points, that is two consecutive points where z reaches the value 0. From the value of T and a given position x_0 of the zero level of the first wave, ge get the next wave by integrating the differential equation

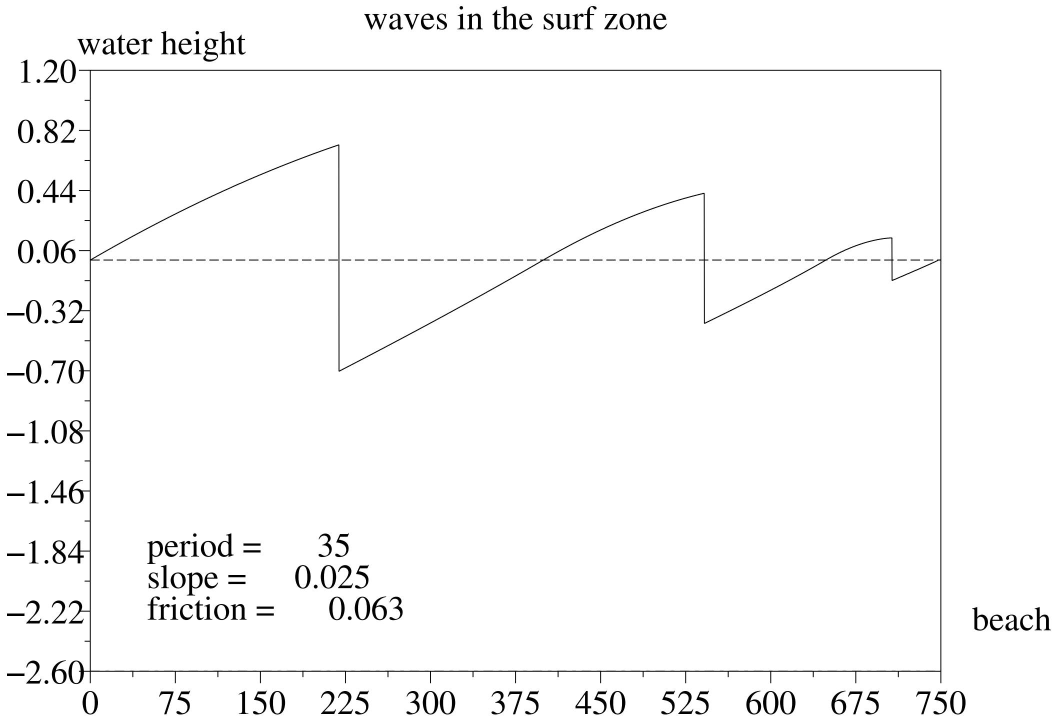
$$x'(t) \ = \ A(x(t)) \ = \ \sqrt{g\ p\ \ (L-x(t))} \tag{6.1}$$

on an interval $[t, t+T]$. We get

$$x(t+T) \;=\; x(t) \;+\; T\,\sqrt{g\;p\;(L-x(t))}\;-\;\frac{T^2\;g\;p}{4}\;,$$

which allows to get the sequence (x_n) of all the zero level points in the surf zone

$$x_{n+1} \;=\; x_n \;+\; T\,\sqrt{g\;p\;(L-x_n)}\;-\;\frac{T^2\;g\;p}{4}\;. \tag{6.2}$$



Next, we consider each interval $[x_n, x_{n+1}]$ and solve twice the differential equation (5.10), the first one denoted z_1 from $x = x_n$ with $z_1(x_n) = 0$ and the second one denoted z_2 from $x = x_{n+1}$ with $z_2(x_{n+1}) = 0$. The solution on $[x_n, x_{n+1}]$ is made of a part of z_1 on the left side of a shock wave and a part of z_2 on the right side of this shock. It remains to find the right position of this shock. We define the continuous function E_* on $[x_n, x_{n+1}]$ by

$$E_*(x) \;=\; q_*(x)^3 \;-\; \left(q_*(x) \;+\; \frac{z_1(x)+z_2(x)}{2}\right)\;(q_*(x)+z_1(x))\;\;(q_*(x)+z_2(x))$$

and from (5.8) the position of the shock corresponds to a root of $E(x)$. This root does exist between x_n and x_{n+1} since $E_*(x_n) > 0$ and $E_*(x_{n+1}) < 0$. We denote $x_{n+\frac{1}{2}}$ this root. We get the final wave on $[x_n, x_{n+1}]$ by taking $z(x) = z_1(x)$ on $\left[x_n, x_{n+\frac{1}{2}}\right]$ and $z(x) = z_2(x)$ on $\left[x_{n+\frac{1}{2}},\; x_{n+1}\right]$. It remains to concatenate these waves on $[x_0, x_1]$, $[x_1, x_2]$, ... to get the wave train until some interval $[x_n, x_{n+1}]$, where the last point x_{n+1} corresponds to the last value for which the inequality

$$x_{n+1} < \;L\; - \;\frac{T^2 gp}{16}$$

holds. The picture above presents such a train of waves. The shapes are computed from the differential equations (5.10) using the Euler method in each direction from each level point x_n and the shock positions are valuted as roots of $E_*(x)$ by using a dichotomy technique.

The choice of q_* as in (5.2) is an academic one. In [11] the author shows that the mean level in this zone does not correspond exactly to the horizontal line as the level at rest. He says that it is in fact an increasing line as we approach the shore. This will correspond to use in (5.2) a parameter $\overline{p} > p$ instead of p. We also remark than the friction coefficient k may be generalized easily in a parameter $k(q)$, without more difficulties in the computations.

7 A two dimension example: the hurricanes

We propose here a new application of this notion in a two dimension space, concerning the hurricanes or cyclones which was first mentioned in [6] . More physical descriptions of the phenomenon are to be found on the net, see for example [10] , [12] , [13]

As in (2.12), we denote by u and v the two horizontal componants of the velocity, $\omega = \frac{\pi\ sin\lambda}{21600}$ is the Coriolis coefficient, where λ is the latitude of the place, $\lambda > 0$ in the northern hemisphere and $\lambda < 0$ in the southern hemisphere, $\mu \geq 0$ is a friction coefficient and $\sigma_* \geq 0$ is a suction coefficient. This suction coefficient has no effect outside the hurricane and simulates the suction due to the vertical componant of the velocity which is not taken explicitly in account here. The gradient of pressure $(p_x,\ p_y)$ is supposed to be a datum here, given from the meteorological previsions or measures. The model is the following one

$$\begin{array}{l} u_t \;+\; u\,u_x \;+\; v\,u_y \;+\; p_x \;+\; \mu\,u \;-\; \omega\,v \;=\; \sigma_*\;(u - u_*) \\ v_t \;+\; u\,v_x \;+\; v\,v_y \;+\; p_y \;+\; \mu\,v \;+\; \omega\,u \;=\; \sigma_*\;(v - v_*) \end{array} \tag{7.1}$$

where

$$u_* \ = \ - \ \frac{\mu \ p_x \ + \ \omega \ p_y}{\mu^2 \ + \ \omega^2} \quad , \quad v_* \ = \ \frac{\omega \ p_x \ - \ \mu \ p_y}{\mu^2 \ + \ \omega^2} \tag{7.2}$$

The wind of velocity field (u_*, v_*) is known as the "trade wind" or the "alizés".

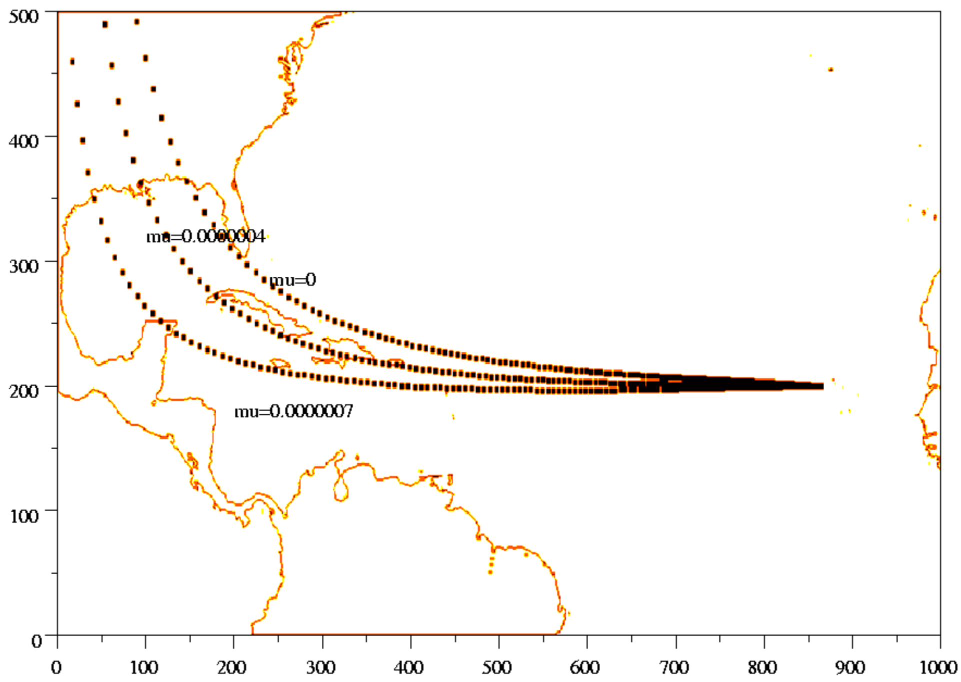
By writing $p_x = -\mu u_* + \omega v_*$ and $p_y = -\omega u_* - \mu v_*$, the system becomes

$$\begin{aligned} u_t \ + \ u \ u_x \ + \ v \ u_y \ &= \ (\sigma_* - \mu)\,(u - u_*) \ + \ \omega\,(v - v_*) \\ v_t \ + \ u \ v_x \ + \ v \ v_y \ &= \ (\sigma_* - \mu)\,(v - v_*) \ - \ \omega\,(u - u_*) \end{aligned} \tag{7.3}$$

where the source term is zero when $u = u_*$ and $v = v_*$. By the way, a source wave is also a solution to the associated linear system

$$u_t \ + \ u_* \ u_x \ + \ v_* \ u_y \ = \ 0 \ \ , \ \ v_t \ + \ u_* \ v_x \ + \ v_* \ v_y \ = \ 0 \ . \tag{7.4}$$

The picture below shows different hurricanes tracks corresponding to different value of the friction parameter μ with a pressure field given by the Azores Anticyclone in the Atlantic Ocean.



These tracks are the characteristics paths of (7.4), that is the solutions of $x'(t) = u_*(x(t), t)$ and $y'(t) = v_*(y(t), t)$ computed by using the Euler method. We notice that the track is very sensitive with repect to the value of μ. For $\mu = 0$ the track follows an isobar of the Azores Anticyclone, and for $\mu > 0$ the track is deviated towards the left of this track. The hurricanes born near the Cape Verde Islands are sent this way towards the Carribean Islands and the south coast of the United States. Other track simulations are given in [14].

We consider now the shock waves. Since the system has a non conservative form, a Rankine Hugoniot condition may be obtained by using the generalized functions theory (see [4] for details). A shock wave for the system (2.1) corresponds to a discontinuity curve of the form $S(x - \frac{u_1 + u_2}{2} t\ ,\ y - \frac{v_1 + v_2}{2} t) = 0$ where (u_1, v_1) and (u_2, v_2) are the values on both sides of the discontinuity. In the particular case where the support of the discontinuity is a circle of radius R and center $(x_0 + u_* t,\ y_0 + v_* t)$ for a given point $(x_0,\ y_0)$ and taking $u_1 = u_*,\ v_1 = v_*$ on one side and $u_2 = \gamma u,\ v_2 = \gamma v$ on the other side, as the trace of the solution, that is

$$\left(x - \frac{u_* + \gamma u}{2} t - x_0 \right)^2 + \left(y - \frac{v_* + \gamma v}{2} t - y_0 \right)^2 - R^2 = 0,$$

the Rankine Hugoniot condition reads

$$(\gamma u - u_*)\ (x - u_* t - x_0)\ +\ (\gamma v - v_*)\ (y - v_* t - y_0)\ =\ 0. \tag{7.5}$$

This imposes the discontinuity jump

$$([u], [v]) = (\gamma u - u_*, \gamma v - v_*) \tag{7.6}$$

to be orthogonal to the radial direction. The shock velocity is still equal to (u_*, v_*) which means that the shock wave is a contact discontinuity. We shall see in the following that the edge of the hurricane may be such a discontinuity, with (u_*, v_*) as the external velocity value. We have also, and that is the most important, the eye wall described by such a shock wave, with (u_*, v_*) as the velocity inside the eye, and a strong value $(\gamma u, \gamma v)$ on the other side of the eye wall. The way the velocity behaves between the external edge and the eye wall corresponds to a bifurcation solution of (7.3).

Using the time t as a parameter, the characteristics system associated with (7.3) reads

$$\begin{array}{rclrcl} x'(t) & = & u(t)\ , & u'(t) & = & (\sigma_* - \mu)\,(u(t) - u_*)\ +\ \omega\,(v(t) - v_*) \\ y'(t) & = & v(t)\ , & v'(t) & = & (\sigma_* - \mu)\,(v(t) - v_*)\ -\ \omega\,(u(t) - u_*)\ . \end{array} \tag{7.7}$$

We consider as a reference trajectory the one which starts from $x - u_* t = R,\ y - v_* t = 0$ where R is the radius of the hurricane. We notice that the integration of the characteristics system leads to an explicit expression of the solution

$u = (\sigma_* - \mu)\,(x - u_* t) + \omega\,(y - v_* t) + u_*,\ \ v = -\omega\,(x - u_* t) + (\sigma_* - \mu)\,(y - v_* t) + v_*.$

We use the Lagrangian variables $X = x - u_* t,\ Y = y - v_* t,\ U = u - u_*,\ V = v - v_*$ and we look for a solution of the form

$$\begin{pmatrix} U \\ V \end{pmatrix} = a(\xi) \begin{pmatrix} X \\ Y \end{pmatrix} + b(\xi) \begin{pmatrix} -Y \\ X \end{pmatrix}$$

where $\xi = \sqrt{X^2+Y^2}$. This way $a(\xi)\xi$ stands for the radial componant of the velocity and $b(\xi)\xi$ for the transverse componant. The Rankine Hugoniot condition reduces to impose the quantity $a(\xi)\xi^2$ to be continuous, which means that the only possible discontinuity for $a(\xi)$ will stay at $\xi = 0$ and that any discontinuity of the transverse componant $b(\xi)\xi$ is allowed.

We suppose for simplicity u_* and v_* constant. After plugging (3.7) into (2.1) and some computations, we get the following differential system as a compatibility condition

$$a\, a'\, \xi \;+\; a^2 \;-\; b^2 \;-\; (\sigma_* - \mu)\, a \;+\; \omega b \;=\; 0,\; a\, b'\, \xi \;+\; 2\, a\, b \;-\; (\sigma_* - \mu)\, b \;-\; \omega a \;=\; 0\,. \quad (7.8)$$

We look for the non zero solutions of this system, which are the bifurcation solutions. In the very particular case where $\mu = 0,\ \omega = 0$, a bifurcation solution reads $a(\xi) = \sigma_* \left(1 - \frac{R}{\xi}\right), b(\xi) = 0$. This is an immediate generalization of the solution of one dimension example given in Section 2, and it can be generalized easily to the three dimension case. This can be seen as a kind of "black hole" (see (2.10), (2.11)).

In the case of a hurricane, bifurcation solutions are obtained by writing $z(\xi) = -\frac{b(\xi)}{a(\xi)}$ in (7.8) which leads to the system

$$\frac{d}{d\xi}\,(a(\xi)\;\xi) \;=\; z(\xi)^2 a(\xi) \;+\; (\sigma_* - \mu) \;+\; \omega z(\xi)\;,\; \frac{d}{d\xi}\,(b(\xi)\;\xi) \;=\; -b(\xi) \;+\; \omega \;-(\sigma_* - \mu)\, z(\xi) \quad (7.9)$$

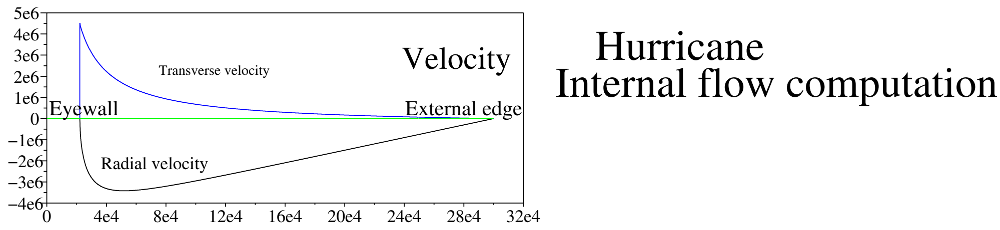
that we solve by integrating by starting from $\xi = R$ with $a(R) = 0,\ b(R) = 0$ and $z(R) = z_0$, an incidence value to be given. It is interesting to notice that the sign of $a'(\xi)$ will change and that $a(\xi)$ will be zero for some value $\xi_0 < R$. When $\xi_0 > 0$ it corresponds to the eyewall. At this point, since $b >> \omega$ in practice, we must have from (7.9) an infinite slope, that is $\lim_{\xi \longrightarrow \xi_0} a'(\xi) = \infty$ and $\lim_{\xi \longrightarrow \xi_0} a(\xi)a'(\xi)$ bounded.

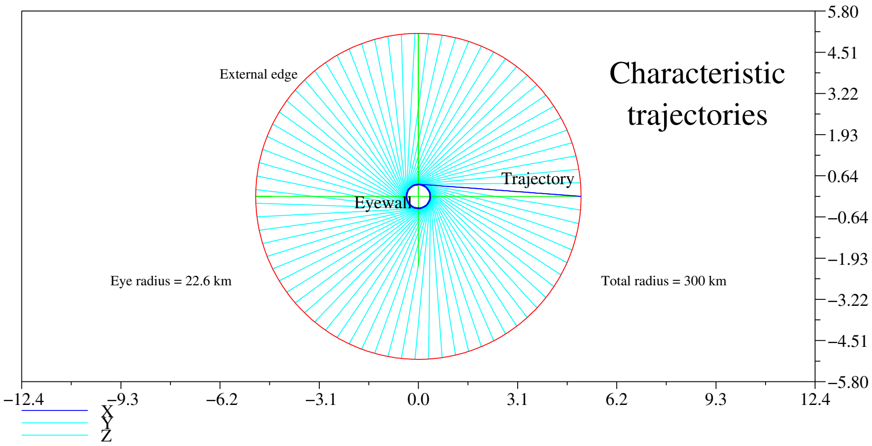
With the same argument, $\lim_{\xi \longrightarrow \xi_0} a(\xi)b'(\xi)$ is also bounded and $\lim_{\xi \longrightarrow \xi_0} b'(\xi) = \infty$.

The computation of the characteristics trajectories may be performed either by using the characteristics equations (7.7) or by using the non linear system (7.8) or (7.9). The first case is more adapted to the case when a shock is present on the edge of the hurricane and the second one is to be used when there is no external shock, in order to have a bifurcation solution.

We present three results. The first one in the first picture corresponds to a hurricane with a small incidence entry and no external shock. We have taken $R = 300km$, and $z_0 = 0.075,\ \mu = 0,\ \sigma_* = 15,\ \omega = 0.0001$. The radial and transverse velocity componants are given above the picture of the hurricane with a selection of characteristics trajectories.

We see that an eye wall is found and that these trajectories come to the eyewall and turn around in the counter clockwise sense since omega is positive (northern hemisphere). The computations are done by solving (7.9) with the Euler method, which explains a large value of the velocity near the eyewall. A more accurate method will reduce this, and this will be also reduced by the natural



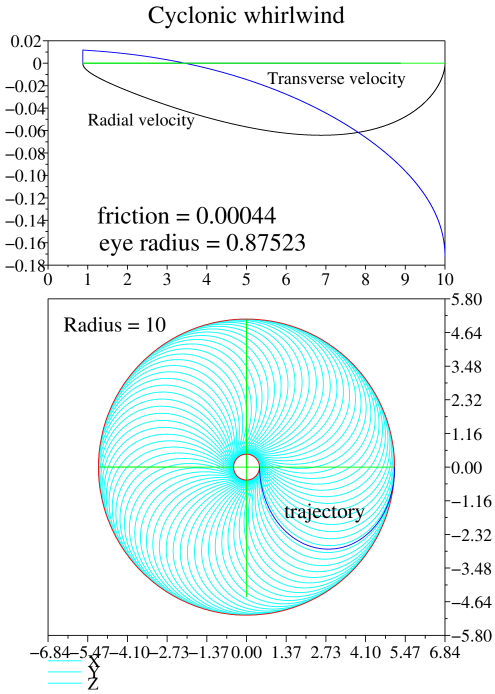


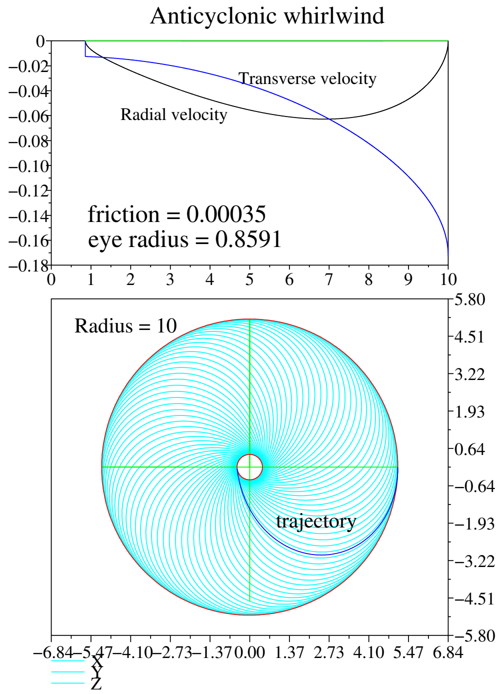
viscosity which is not taken in account here. On the other hand this experiment proves that even with an order one scheme the eye is captured.

The two next pictures correspond to two smallest whirlwinds, like tornadoes or waterspouts, starting with an external shock on the edge. Here $R = 10m$, $k = 0$, $\omega = 0.002$. We have tested two values of the friction, and found that for $\mu = 0.00044$ the flow around the eye is a counter clockwise one and for $\mu = 0.00035$ it is a clock wise one. This underline that the friction term plays some role in the way the flow reaches the eye. There is a value between these two ones for which the eye collapses. Here an explicit calculus of the differential system (7.7) has been performed.

We remark that the influence of the friction parameter is very important since it determines the way the flow turns around the eye for a whirlwind or can make a strong deviation of the track of a hurricane. It influences also the size of the eye. The parameters have been valued in such a way to present examples with a realistic size or behaviour. They are not obtained from experimental measurements.

We also observe that no mass conservation equation has been taken in account. As a matter of fact the mass of air or vapor is not conserved in a two dimension model since it is not the average approximation of a three dimension problem but only a simple model which describes the behaviour near the ground or the sea. Nevertheless the mass of air and vapor has to be conserved in a three dimension model.





8 Conclusion

We have presented the notion of source waves by using several examples in environmental science. Other examples are also possible as for avalanche modeling, dam breaks on a slope, flows in a pipe, etc... since those applications use models corresponding to nonlinear hyperbolic systems with non linear source terms. However, the examples we have proposed always correspond to simple situations, as a constant slope for example in hydraulics. For more complex applications we can use locally our results. This is the case in our computation of the hurricanes tracks since ω, p_x and p_y need to be updated at any time.

The particular solutions we have presented are difficult to obtain with a standard numerical scheme since for example a splitting scheme will destroy any relation between the flux and the source term, whose balance absolutely needs to be maintained. A Godunov scheme will reduce the shocks even in the one dimension case of a scalar equation. On the other hand, they correspond to useful example for checking the quality of a numerical scheme. Some of these ideas were already present in the references [3] and [5], and used to derive some numerical schemes with a high quality of stability.

Acknowlegments: we thank Professor J.A.Marti (see [15]) and others colleagues at the Antilles Guyane University in Pointe-à-Pitre, Guadeloupe for very helpful informations on hurricane descriptions. We also thank Ph. Bonneton(Bordeaux University) for instructive discussions about water waves.

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