New scenario for transition to slow 3-D turbulence

Jakov Foukzon
Israel Institution of Technology

aguidance@excite.com

Abstract. Numerical-analytical study of the three-dimensional nonlinear stochastic partial differential equation, analogous to that proposed by V.N.Nikolaevskii [Recent Advances in Engineering Science (Springer -- Verlag, Berlin. 1989)] to describe longitudinal seismic waves, ispresented. The equation has a threshold of short-wave instability and symmetry, providing long-wave dynamics. Proposed new mechanism for quantum chaos generating in nonlinear dynamical systems. The hypothesis is said, that physical turbulence could be identified with quantum chaos of considered type.

Key words: 3D turbulence, internal path integral, chaos, quantum chaos.

Introduction.In the present work a non-perturbative approach to the studying of problem of quantum chaos in dynamical systems with infinite number of degrees of freedom is proposed. Proposed approach also allows to estimate the influence of additive (thermal) fluctuations on the processes of formation of developed turbulent modes in essentially non-linear processes like electro-convection and other. A principal role of fluctuations on the dynamics of some types of dissipative systems in the proximate environs of derivation rapid of a short-wave instability was ascertained. General results (**Theorem 2**) are illustrated on example of model system:

$$\frac{\partial}{\partial t}u_\eta + \Delta\left[\varepsilon - (1+\Delta)^2\right]u_\eta + \delta_1\left(\frac{\partial}{\partial x_1}u_\eta\right)u_\eta +$$

$$+\delta_2\left(\frac{\partial}{\partial x_2}u_\eta\right)u_\eta + \delta_3\left(\frac{\partial}{\partial x_3}u_\eta\right)u_\eta +$$

$$+au_\eta^3 + bu_\eta^2 + f(\bar{x},t) - \sqrt{\eta}\,\check{W}(\bar{x},t) = 0,$$

$$u_\eta(\vec{x},0) = 0,$$

$$\check{W}(\bar{x},t) = \frac{\partial^4}{\partial t\partial x_1\partial x_2\partial x_3}W(\bar{x},t), \tag{1.1}$$

Such an approximation gives a powerful analysis means of the processes of turbulence conception, based on the classical theory of chaos of the finite-dimensional classic systems. However, as it good known, such an ap-proximation is correct only in a phase of turbulence conception, when relatively small number of the degrees of freedom excites. At the present time it is gener-ally recognized that turbulence in its developed phase has essentially singular spatially-temporal structure. Such a singular conduct is impossible to describe adequately by the means of some model system of equations of a finite dimen-sionality. In this point a classical theory of chaos is able to describe only small part of turbulence phenomenon in liquid and another analogous systems. The results of numerical modeling of super-chaotic modes, obtained in the present work, allow to put out a quite probable hypothesis: developed turbulence in the real physical systems with infinite number of degrees of freedom is a *quantum super-chaos*, at that the quantitative characteristics of this super-chaos is completely determined by non-perturbative contribution of additive (thermal) fluctuations in the corresponding classical system dynamics.

General results. We study the $r-$ dimensional differential equation analogous proposed by Nikolaevskii [1] to describe longitudinal seismic waves:

$$\frac{\partial u}{\partial t}+\Delta\left[\varepsilon-(1+\Delta)^2\right]u+u\sum_{i=1}^{r}\delta_i\frac{\partial u}{\partial x_i}+au^3+$$
$$+bu^2+f(\vec{x},t)=0,$$
$$u(\vec{x},0)=0,$$
(2.1)

$$a\geq 0,\Delta=\sum_{i=1}^{r}\frac{\partial^2}{\partial x_i^2},x\in R^r,$$

($r-$ dimensional N -- model), and stochastic differential equation

$$\frac{\partial}{\partial t}u_\eta+\Delta\left[\varepsilon-(1+\Delta)^2\right]u_\eta+$$
$$+u_\eta\sum_{i=1}^{r}\delta_i\frac{\partial u_\eta}{\partial x_i}+au_\eta^3+bu_\eta^2+f(\vec{x},t)- \qquad (2.2)$$
$$-\sqrt{\eta}\,\check{W}(x,t)=0,$$

$$u_\eta(\vec{x},0)=0,$$
$$a\geq 0,\eta\approx 0,\Delta=\sum_{i=1}^{r}\frac{\partial^2}{\partial x_i^2},x\in R^r,$$
$$\check{W}(x,t)=\frac{\partial^{r+1}}{\partial t\partial x_1\partial x_2\ldots\partial x_r}W(x,t),$$

$\check{W}(x,t)-$ white noise, (quantum N -- model).

Remark1. If $f(\vec{x},t)=f(\vec{x}),$ then equation (2.2) is possible to consider as a result of application of the formal procedure of stochastic quantization [2] to the equation of type

$$\Delta\left[\varepsilon-(1+\Delta)^2\right]u+$$
$$+u\sum_{i=1}^{r}\delta_i\frac{\partial u}{\partial x_i}++au^3+bu^2+f(\vec{x},t)=0,$$

(static N-model) that is to say as a corresponding Euclidian equation of Langeven. By another words, we can consider the quantum N-model as a result of quantization of the static N-model. In [3] is executed the numerical study one-dimensional nonlinear partial differential equation

$$\frac{\partial w}{\partial t}+$$
$$+\frac{\partial^2}{\partial x^2}\left[\varepsilon-\left(1+\frac{\partial^2}{\partial x^2}\right)\right]w+$$
$$+\left(\frac{\partial^2 w}{\partial x^2}\right)=0,w(x,0)\approx 0,$$

which is the equivalent form of 1-dimensional N-model (2.1) ($2\partial w/\partial x\stackrel{\Delta}{\equiv}u$) with parameters $a=0,\ b=0,\ f(x)\equiv 0$ The numerical study [3] $0<\varepsilon<<1$ shows that any small-amplitude initial spatial distribution of the order parameter $w(x,0)$ evolves, after a certain transient period, into one or another time-dependent asymptotic regime. All these regimes are characterized by excitation of bunches of modes from narrow subbands centered around the points $k=\pm n,$ where the integer n may be regarded as a number of the corresponding subband. Amplitudes of the modes fall off rapidly to zero with increase in both the deviation of k from the center of each subband, at fixed $n,$ and the number $n,$ at the fixed deviation. The question about a role of additive (thermal) fluctuations in the problems of the present type, considered in [3], is *exceptionally important*. In [3] M. Tribelsky emphasizes the circumstance, that the determining role of the fluctuations in environs of the phase transition's points is well-known, and in a certain situation the presence of fluctuations can change a kind of the phase transition from the second to the first one. Nevertheless, in [3] the supposition is said, that in connection with the circumstance, that in dissipative systems the forming structures always have a macroscopic scale, the question about the influence of fluctuations on the dynamics of a system loses a real purport. In actual fact such a supposition is deeply erroneous. On the mathematical language such a supposition is equivalent to the following utmost equality

$$\limsup_{\eta\to 0}E\left[u_\eta(\vec{x},t,\omega)-u(\vec{x},t)\right]^2=0. \qquad (2.3)$$

From the general theory of stochastic differential equations it is known, that equality (2.3) is wittingly executed only with certain limitations on nonlinear terms of the corresponding equations. For equation (2.2) such limitations are not executed. On the physical language, on the strength of remark 1, equality (2.3) means, that in quantum N-model the quantum chaos is absent in Euclidian district. It should be noted, that a problem of the quantum chaos is usually presented for the systems with a finite number of degrees of freedom, and lies in a search of the constructive methods of comparison of the chaotic dynamic of a classical system and its quantum analogue with $\hbar \to 0$. If the stochastic quantization is used, the present comparison is executed with $\eta \to 0.$ Since the classical system (1.1) has a complicated chaotic dynamics, then, proceeding from experience of the studies of a problem of quantum chaos for the systems with a finite number of degrees of freedom, it is completely impossible to suppose, that the quantum effects in quantum N-model will not influence essentially on its quasiclassical dynamics.

Theorem1. For all solutions of equation (2.2) and parameters

$\lambda, \in R, t \in R_{+}, \vec{x} \in R^{r}, a \in (0,\infty)$:

(1)

$$\Re(\vec{x},t,a,\lambda)=0 \Rightarrow \Rightarrow \liminf_{\eta\to 0} E\left[u_{\eta}(\vec{x},t,\omega)-\lambda\right]^{2}=0, \qquad (2.4)$$

(2)

$$\Re(\vec{x},t,0,\lambda)=0 \Rightarrow \Rightarrow \liminf_{a\to 0}\left[\liminf_{\eta\to 0} E\left[u_{\eta}(\vec{x},t,\omega)-\lambda\right]^{2}\right]=0,$$

where $\Re(\vec{x},t,a,\lambda)$ solution of the linear partial differential equation

$$\frac{\partial\Re}{\partial t}+\Delta\left[\varepsilon-(1+\Delta^{2})\right]\Re+\lambda\sum_{i=1}^{r}\delta_{i}\frac{\partial\Re}{\partial x_{i}}+ +(3a\lambda^{2}+2b\lambda)\Re+f(\vec{x},t)+a\lambda^{3}+ +b\lambda^{2}=0, \qquad (2.5)$$

$$\Re(\vec{x},0,\lambda)=-\lambda.$$

Theorem 2. Let B is separable Banach space, $W(t,\omega)$ is so a standard Wiener process that $\forall t,\omega, W(t,\omega)\in B.$ Lets consider the stochastic differential equation

$$\tfrac{d}{dt}u_{\eta}(t)=Lu_{\eta}(t)+\Im(u_{\eta}(t),t)u_{\eta}(t)+ +\sqrt{\eta}\,\dot{W}(t),$$

$$\dot{W}(t)=\tfrac{d}{dt}W(t), \quad u_{\eta}(0)=0,$$

or in the integral form

$$u_{\eta}(t,\omega)=\int_{0}^{t}(Lu_{\eta}(s,\omega))ds+ +\int_{0}^{t}(\Im(u_{\eta}(s,\omega),s)u_{\eta}(s,\omega))ds+ +\sqrt{\eta}\int_{0}^{t}dW(s,\omega)$$

where L is linear operator $L: B\to B,$ and $\Im(u_{\eta})$ is nonlinear operator $\Im(u_{\eta}): B\to B.$ Let the following additional conditions are executed:

(1) for any $\eta<1$ equation (**E**) has the only strong solution $u_{\eta}(t,\omega), t\in[0,\infty).$

(2) there is such a basis $\{e_{k}\}$ in $B,$ that for all $t\in[0,\infty)$:

$$lim_{n\to\infty}\int[\|u_{\eta}(t,\omega)-u_{\eta,n}(t,\omega)\|_{B}^{2}]d\omega=0,$$

where $u_{\eta,n}(t,\omega)$ is the solution of equation

$$u_{\eta,n}(t,\omega)=\int_{0}^{t}(P_{n}Lu_{\eta,n}(s,\omega))ds+ +\int_{0}^{t}(P_{n}\Im(u_{\eta,n}(s,\omega),s)u_{\eta,n}(s,\omega))ds+ +\sqrt{\eta}\int_{0}^{t}dw_{n}(s,\omega),$$

P_n is the operator of the projection on subspace in B, which is generated by $\{e_1, \ldots, e_n\}$, $w_n(s,\omega) = P_n W(s,\omega)$.
Then

$$\underline{lim}_{\eta\to 0}\langle u_\eta(t,\omega)\rangle \leqslant |\Re(t)| exp(\psi(t)),$$
$$\forall\, t \in [0,\infty),$$

where $\Re(t)$ is the solution of differential equation in Banach space B

$$\frac{\partial}{\partial t}\Re(t) = L(t,\Re(t)), \Re(0) = 0.$$

Theorem 2 has completely general character and allows to obtain the result analogous to theorem I for sufficiently wide class of PDE's, including Navier-Stokes equations

Theorem 1 allows to study by *nonperturbative way* the influence of thermal additive fluctuations on classical dynamics, which in the considered case is described by equation (2.1). In the case when $a > 0$, $a >> 1$, the influence of fluctuations is of no special importance and stochastic dynamics almost does not differ from classical dynamics. However if $a \approx 0$, then distinctions have so radical character, then in initial system a new type of *physical chaos* appears. We shall call such a chaos the **quasideterminate chaos (QD-CHAOS)** meaning by this the circumstance, that such a chaos appears in the result of utmost transformation: $\eta \to 0$. A concrete structure of this chaos is determined by solutions variety of *permessive equation*

$$\Re\left(\vec{x},t,\lambda\right) = 0 \qquad \textbf{(PE)}$$

From another side equation (I) determines by the only way some many-valued function $\lambda(\vec{x},t)$, which is the main constructive object, determining the characteristics of quantum chaos in the corresponding model of Euclidian field theory.
The mathematical character of *quasidetermined chaos* is sufficiently simple and constrained with a "very strong" dependence of the trajectories of process $u_\eta(\vec{x},t,\omega)$ from small parameter η, where η is the fluctuations intensity. In this point a character of *quasideterminated chaos* is analogous to a character of usual *dynamical chaos,* which is the consequence of a "very strong" dependence of the dynamics from initial conditions. However these two kinds of chaos are principally differ one from another, because *quasidetermined chaos* has essentially singular character, that on the mathematical language is expressed by the condition

$$mes\{(\vec{x},t) \in R^r \times R_+ | \neg\exists lim_{\eta\to 0}\langle u_\eta(\vec{x},t)\rangle\} > 0$$

QDC-*quasidetermined chaos* condition.
Condition (**QDC**) is obviously equivalent to the condition that $\lambda(\vec{x},t)$ is many-valued function ($card\left\{\lambda(\vec{x},t)\right\} > 1$, where $\left\{\lambda(\vec{x},t)\right\}$ is the set of all the values of function $\lambda(\vec{x},t)$ in point $(\vec{x},t)$) on the set of positive measure:

$$mes\left\{(\vec{x},t) \in R^r \times R_+ \mid card\left\{\lambda(x,t)\right\} > 1\right\} > 0 .$$

Definition1.

(a) If for all points $(\vec{x},t) \in R^r \times R_+$:

$$\exists \lim_{\eta\to 0} E[u_\eta^2(\vec{x},t,\omega)] = u^2(\vec{x},t),$$

we say that $u(\vec{x},t) -$ **QD-solution** (quasi determined solution) of the eq.(2.2).

(b) If

$$mes\{(\vec{x},t) \in R^r \times R_+ | \neg\exists lim_{\eta\to 0}\langle u_\eta(\vec{x},t)\rangle\} > 0,$$

we say that quantum N-model have the Euclidian quantum chaos (**EQC**).

Definition2. For each point $(\vec{x},t) \in R^r \times R_+$ lets define the set $\left\{\overset{\otimes}{\Re}\left(\vec{x},t\right)\right\}$ by the means of the following condition:

$$\forall\lambda\left[\lambda \in \left\{\overset{\otimes}{\Re}\,(\vec{x},t)\right\} \Leftrightarrow \Re(\vec{x},t,\lambda) = 0\right],$$

and define the many-valued function

$$\overset{\otimes}{\Re}(\vec{x},t):\ R^r \times R_+ \to 2^R,$$

$$\overset{\otimes}{\Re}(\vec{x},t) \overset{\Delta}{=} \left\{\overset{\otimes}{\Re}(\vec{x},t)\right\}.$$

We say that $\overset{\otimes}{\Re}(\vec{x},t)$ *quasidetermined chaotic solution* (**QD-chaotic solution**) of the quantum N-model.

For numerical study of the QD-chaotic solutions of the quantum N-model, we have the equation $\Re(x,t,\lambda(\vec{x},t)) = 0.$

Theorem3. For all volumes of parameters $r \in N, \varepsilon, a \ll 1, b, \sigma \neq 0, \vec{p} \in R^r,$ $f(\vec{x},t) = \sigma \times \sin(\langle \vec{p}, \vec{x} \rangle), r$ -- dimensional quantum N-model (eq.(2.2)) has the **QD-**chaotic solutions.(In these cases **QD**-solutions of the eq.2 evidently is absent).

Definition3. (1):

$$u_+(\vec{x},t) \overset{\Delta}{=} \limsup_{\eta \to 0} E\left[u_\eta(\vec{x},t,\omega)\right],$$

$$u_-(\vec{x},t) \overset{\Delta}{=} \liminf_{\eta \to 0} E\left[u_\eta(\vec{x},t,\omega)\right],$$

$$u^\vee(\vec{x},t) \overset{\Delta}{=} u_+(\vec{x},t) - u_-(\vec{x},t),$$

$u_+(\vec{x},t) -$ up border of the quantum chaos in Euclidian quantum N-model, $u_-(\vec{x},t) -$ down border of the quantum chaos in Euclidian quantum N-model, $u^\vee(\vec{x},t) -$ extent of quantum chaos in Euclidian quantum N-model.

(2): If

$$\limsup_{t \to \infty} \left(\sup_{\vec{x}} u^\vee(\vec{x},t) \right) \overset{\Delta}{=} u^\vee_\infty < \infty,$$

we shall say, that Euclidian quantum N-model has quantum chaos of the finite extent, otherwise we shall say, that Euclidian quantum N-model has unlimitedly growing quantum chaos.

(3):

$$\overset{\otimes}{\Re}_+(\vec{x},t) \overset{\Delta}{=} \max\left\{\overset{\otimes}{\Re}(\vec{x},t)\right\},$$

$$\overset{\otimes}{\Re}_-(\vec{x},t) \overset{\Delta}{=} \min\left\{\overset{\otimes}{\Re}(\vec{x},t)\right\},$$

$$\overset{\otimes\,\vee}{\Re}(\vec{x},t) \overset{\Delta}{=} \overset{\otimes}{\Re}_+(\vec{x},t) - \overset{\otimes}{\Re}_-(\vec{x},t),$$

$$\limsup_{t \to \infty}\left(\sup_{\vec{x}} \left(\overset{\otimes}{\Re}(\vec{x},t) \right)\right) \overset{\Delta}{=} \overset{\otimes\,\vee}{\Re}_\infty(\varepsilon) = \overset{\otimes\,\vee}{\Re}_\infty .$$

Evidently: (Theorem1) $\Rightarrow \overset{\otimes}{\Re}_+(\vec{x},t) \leqslant u^\vee(\vec{x},t).$

Theorem 4. (criterion of quantum chaos in Euclidian quantum N-model).

If :

(1) $mes\left\{(\vec{x},t) \mid \overset{\otimes}{\Re}(\vec{x},t) > 0\right\} > 0$

Euclidian quantum N-model has the quantum chaos.

If :

(2) $\overset{\otimes\,\vee}{\Re}_\infty = \infty,$

Euclidian quantum N-model has unlimitedly growing quantum chaos.

Definition4.

$N(\vec{x},t) = card\left\{\overset{\otimes}{\Re}(\vec{x},t)\right\}, N(\vec{x},t) -$

numerical function of quantum chaos in Euclidian quantum N-model.

$$\begin{aligned}&\frac{\partial u_\eta}{\partial t} + \frac{\partial^2}{\partial x^2}\left[\varepsilon - \left(1 + \frac{\partial^2}{\partial x^2}\right)\right] u_\eta + \\ &+ \delta u_\eta \frac{\partial u_\eta}{\partial x} - \sigma \sin(px) - \sqrt{\eta}\, \dot{W}(x,t) = 0,\end{aligned} \tag{2.11}$$

$$u_\eta(x,0) = 0,$$

$$\begin{aligned}&\frac{\partial \Re}{\partial t} + \frac{\partial^2}{\partial x^2}\left[\varepsilon - \left(1 + \frac{\partial^2}{\partial x^2}\right)\right]\Re + \\ &+ \delta\lambda \frac{\partial \Re}{\partial x} - \sigma \sin(px) = 0,\end{aligned} \tag{2.12}$$

$$\Re(x,0,\lambda) = -\lambda.$$

Then we have the permessive equation in form:

$$\Re(x,t,\lambda) = \sigma\left[-\frac{\left[\cos(px)-e^{t\chi(p)}\cos(p(x-\delta\cdot\lambda\cdot t)\right]\omega}{\chi^2(p)+p^2\cdot\lambda^2\cdot\delta^2} - \frac{\left[\sin(px)-e^{t\chi(p)}\sin(p(x-\delta\cdot\lambda\cdot t\right]\cdot\chi(p)}{\chi^2(p)+p^2\cdot\lambda\cdot t}\right] - \lambda,$$

$$\omega = p\cdot\delta\cdot\lambda,$$

$$\chi(p) = p^2[\varepsilon-(p^2-1)^2].$$

The result of calculation of corresponding function $\overset{\otimes}{\Re}\left(\vec{x},t\right)$; see on Fig's.1,2,3,4.

In generally accepted at the present time approach to occurrence of the physical turbulence in the dynamical systems with an infinite number of degrees of free-dom the turbulence is associated with proposed presence of the strange attrac-tors, on which the phase trajectories of dynamical system reveal the known properties of stochasticity: a very high dependence on the initial conditions, which is associated with exponential dispersion of the initially close trajectories and brings to their nonreproduction; everywhere the density on the attractor almost of all the trajectories; a very fast extinction by $\tau\to\infty$ of auto-correlation functions.

The obtained results convincingly show, that by the presence of even the infinitesimal fluctuations the classical trajectories cannot influence on the dynamics of a system with an infinite number of degrees of freedom. A supposition, according to which *the physical turbulence could be identified with quantum chaos* of the considered type, seems sufficiently natural. In order to bring in the characteristics of this turbulence, let's define functions $\overset{\otimes}{\Re}_i(\vec{x},t), i=\overline{1,N(\vec{x},t)}$, which's value in point equals the $i-$ th element of set $\left\{\overset{\otimes}{\Re}\left(\vec{x},t\right)\right\}$, ordered in accordance with increase of its element. Let's define

$$\bar{u}(\vec{x},t)\overset{\Delta}{=}[N(\vec{x},t)]^{-1}\sum_{i=1}^{N(\vec{x},t)}\overset{\otimes}{\Re}_i\left(\vec{x},t\right) \qquad (2.14)$$

The values of the turbulent pulsations we shall define by:

$$\bar{u}'(\vec{x},t)\overset{\Delta}{=}\sqrt{\frac{1}{N(\vec{x},t)}\sum_{i=1}^{N(\vec{x},t)}\left(\overset{\otimes}{\Re}_i(\vec{x},t)-\bar{u}(\vec{x},t)\right)} \qquad (2.15)$$

Let's define also a local auto-correlation function

$$\Phi\left(\vec{x},\tau\right)=\lim_{T\to\infty}\frac{1}{T}\int_0^T\tilde{u}(\vec{x},\tau)\tilde{u}(\vec{x},\tau+t)dt,$$

$$\tilde{u}(\vec{x},t)=\bar{u}(\vec{x},t)-\bar{\bar{u}}(\vec{x}), \qquad (2.16)$$

$$\bar{\bar{u}}(\vec{x})=\lim_{T\to\infty}\frac{1}{T}\int_0^T\bar{u}(\vec{x},t)dt,$$

and normalized local auto-correlation function

$$\Phi^{\circ}(\vec{x},\tau)=\frac{\Phi(\vec{x},\tau)}{\Phi(\vec{x},0)}. \qquad (2.17)$$

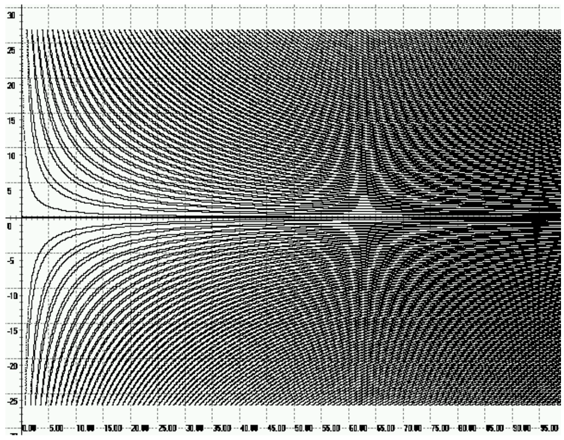

FIG.1. Start of development chaotic regime, in 1-dimensional quantum N-model, $\varepsilon=10^{-7},\sigma=10^2,$

$\delta=1,p=1,t\in[0,100],x=1.$ (scale by axis **X** - 1:1,scale by axis **Y**- 1:4)

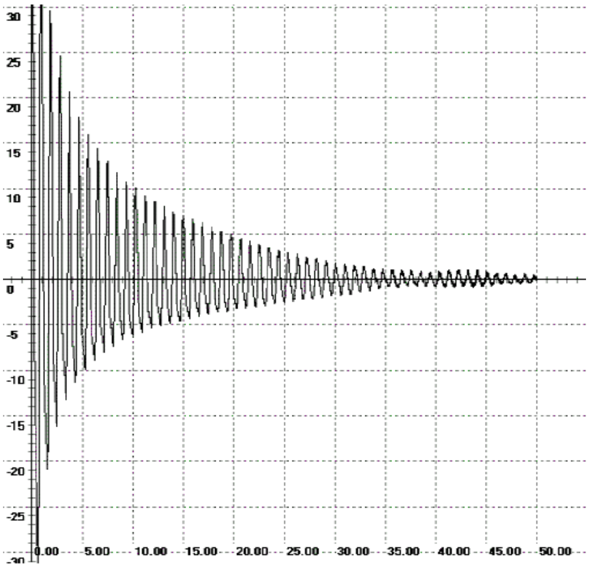

FIG.2. Normalized local auto-correlation function $\Phi^{\circ}(1,\tau), t\in[0,50], \varepsilon = 10^{-7}, \sigma = 10^{2}, \delta = 1,$ $p = 1.$
(scale by axis **X** - 1:1, scale by axis
Y - 1:90).

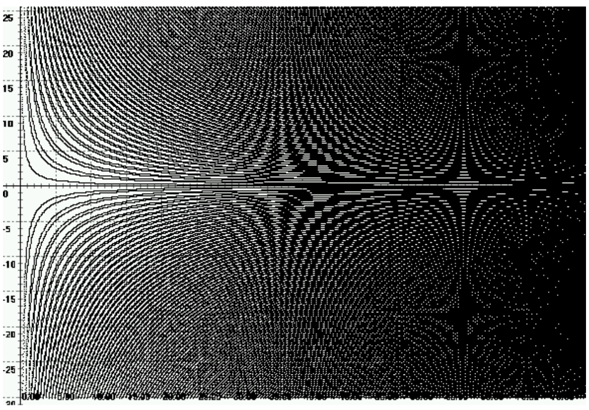

FIG.3. Start of development chaotic regime, in 1-dimensional quantum N-model,
$\varepsilon = 10^{-7}, \sigma = 5\cdot 10^{2}, \delta = 1,$
$p = 1, t\in[0,100], x = 1.$
(scale by axis **X** - 1:1, scale by axis
Y - 1:2).

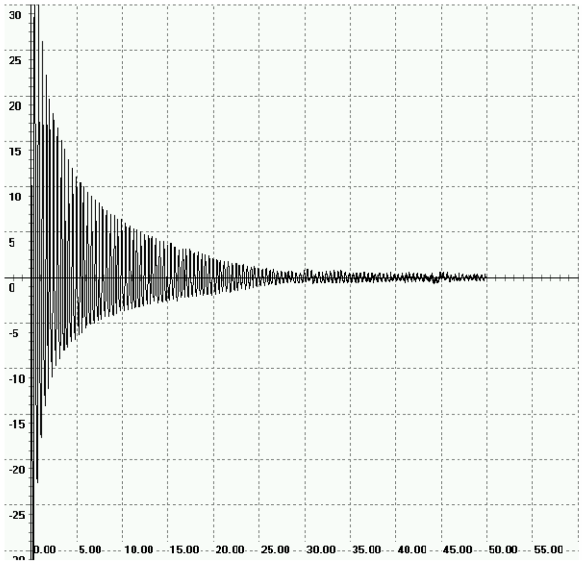

FIG.4. Normalized local auto-correlation
Function
$\Phi^{\circ}(1,\tau), t\in[0,50], \varepsilon = 10^{-7},$
$\sigma = 5\cdot 10^{2}, \delta = 1, p = 1.$
(scale by axis **X** - 1:1 , scale by axis
Y - 1:100).

For research of the question about the presence and the character of
the phase transition it is possible to use equation

$$\Re(\vec{x}, t, \lambda(\varepsilon), \varepsilon) = 0 \qquad (2.18)$$

from which, as a result of differentiation by parameter ε we
obtain the identity

$$\frac{\partial \Re}{\partial \lambda}\frac{d\lambda}{d\varepsilon} + \frac{\partial \Re}{\partial \varepsilon} = 0.$$

(2.19)

where we finally have

$$\frac{d\lambda}{d\varepsilon} = -\left(\frac{\partial \Re}{\partial \varepsilon}\right)\cdot\left(\frac{\partial \Re}{\partial \lambda}\right)^{-1}. \qquad (2.20)$$

From expression (2.20) the function:

$$\Theta_{\pm}(\vec{x}, t) = \left(\frac{d\lambda(\vec{x}, t)}{d\varepsilon}\right)_{\varepsilon=\pm 0} \qquad (2.21)$$

is defined, taking (2.13) into account, by
the straight calculation. It is clear, that

$$\Theta_{-}(\vec{x},t) = \Theta_{+}(\vec{x},t), \forall t < \infty. \qquad (2.22)$$

From other side, correlation

$$\pm\infty \neq \overset{\otimes\vee}{\Re}_{\infty}(-\varepsilon) \neq \overset{\otimes\vee}{\Re}_{\infty}(\varepsilon) = \infty, \varepsilon > 0, \qquad (2.23)$$

is true.

Equations (2.22), (2.23) show, that by $\varepsilon = 0$ in a system is present a soft phase transition from the chaos of the finite extent gives place to the chaos of infinite extent. By the value of critical parameter $\varepsilon = 0$ in system (2.2) by $t \to \infty$ the following asymptotic condition is executed:

$$\lim_{t\to\infty} N(\vec{x},t) \to \infty. \qquad (2.24)$$

Thus, by $\varepsilon > 0, a = 0$ system (2.2) possesses by non-physical asymptotic dynamics by $t \to \infty$, which is chaos of infinite extent (2.23).

Conclusion. Thus we come to the following *completely clear* conclusions which are in obvious contradiction with conclusions made by Tribelsky [2],[3] on the basis of a strict numeral modeling:

(1) Model system (2.25) considered by Tribelsky [3] and possessing by *hidden additional symmetry* concerning transformations group $u_{\eta}(\vec{x},t) \to u_{\eta}(\vec{x},t) + \xi, \xi \in \mathbb{R}$:

$$\frac{\partial}{\partial t}u_{\eta} + \Delta\left[\varepsilon - (1+\Delta)^2\right]u_{\eta} +$$

$$+ u_{\eta}\sum_{i=1}^{r}\delta_i \frac{\partial u_{\eta}}{\partial x_i} + f(\bar{x},t) - \qquad (2.25)$$

$$-\sqrt{\eta}\,\overset{\vee}{W}(x,t) = 0,$$

$$u_{\eta}(\vec{x},0) = 0,$$

can not be principally a basis for analysis of a physical mechanism generating the slow turbulence in a real non-linear processes like electro-convection and others by $\varepsilon > 0$, because by $\varepsilon > 0, t \to \infty$ the corresponding the corresponding asymtotic dynamics *loses its physical point* [chaos of infinite extent(2.23)] by the presence of additive thermal fluctuations like white noise in a system. (2.23)] by the presence of additive thermal fluctuations like white noise in a system. asymtotic dynamics *loses its physical point* [chaos of infinite extent (2.23)] by the presence of additive thermal fluctuations like white noise in a system.

(2) Model system (2.26) considered by Tribelsky [3] and possessing by *clear additional symmetry* concerning tranformations group $w_{\eta}(\vec{x},t) \to w_{\eta}(\vec{x},t) + \xi, \xi \in \mathrm{R}$:

$$\frac{\partial}{\partial t}w_{\eta} + \Delta\left[\varepsilon - (1+\Delta)^2\right]w_{\eta} + \qquad (2.26)$$

$$+\sum_{i=1}^{r}\delta_i\left(\frac{\partial w_{\eta}}{\partial x_i}\right)^2 + f(\bar{x},t) - \sqrt{\eta}\,\overset{\vee}{W}(\vec{x},t) = 0,$$

$$w_{\eta}(\bar{x},0) = 0,$$

where $W(\vec{x},t)$ is Wiener's process, as a result of transformation $2\partial w/\partial x \overset{\Delta}{\equiv} u$ by a clear way transforms into system **PDE's** which have a structure analogous the equation (2.25). Accordingly, model (2.26) also possesses by *non-physical asymptotic dynamics* by $\varepsilon > 0, t \to \infty.$

(3) In a case $\varepsilon < 0,$ the both (2.25) and (2.26) models possess by the correct asymptotic dynamics by $t \to \infty$.

(4) By the value of critical parameter $\varepsilon = 0,$ in models (2.25) and (2.26) the following asymptotic condition is executed: $\lim_{t\to\infty} N(\bar{x},t) \to \infty,$ if $1 \in Spec[f]$.

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