

A note on the carbuncle in shallow water simulations

Friedemann Kemm

*Institute for Applied Mathematics and Scientific Computing, Brandenburgische Technische Universität
Cottbus, Platz der Deutschen Einheit 1, 03046 Cottbus, Germany*

Abstract

An important problem in gas dynamics simulation is to prevent the carbuncle, a breakdown of discrete shock profiles. We show that for high Froude number, this also occurs in shallow water simulations and give numerical evidence that all cures developed for gas dynamics should also work in shallow water flows.

Keywords: Shallow Water, Carbuncle, High Froude Number

2010 MSC: 35L65, 65M08, 76L05, 76M12

1. Introduction

Starting with the seminal paper by Quirk [1], many investigations of the carbuncle, the breakdown of discrete shock profiles, in gas dynamics and many strategies to suppress it were published, e. g. [2, 3, 4, 5, 6, 7, 8]. Among the classical complete Riemann solvers, i. e. Riemann solvers which explicitly resolve all waves in the Riemann problem, the Roe solver is known to be most prone to that numerical artifact and the Osher solver to be least prone to it. Solvers like HLLE, which does not resolve entropy and shear waves, are known to avoid that effect completely.

The carbuncle is known to be mainly driven by an interplay between one-dimensional shock instabilities like postshock oscillations [4] or jumping shock positions [3] and undamped unphysical vortices along the shock front.

So, there are two major ways to prevent a scheme from producing carbuncle like structures: Adjust the viscosity on strong shocks to stabilize them (like Osher) or adjust the viscosity on entropy and shear waves (like HLLE). Since the Osher scheme cannot completely avoid the carbuncle [9], most cures follow the second way, which in turn offers two major approaches: raise the viscosity on linear waves whenever a strong shock in a neighbouring Riemann problem is detected or raise the viscosity when the residual in the Rankine Hugoniot condition for that wave is large, e. g. [10]. The price

Email address: kemm@math.tu-cottbus.de (Friedemann Kemm)

for the first approach is a nonlocal data treatment, the price for the second is a slight increase of shear viscosity outside of single shear waves.

Since mathematically there are some similarities between the Euler equations of gas dynamics and the shallow water equations [11, 12], and since Denlinger and O’Connell [13] report carbuncle-like instabilities in their simulations, we expect the carbuncle to also appear in shallow water flows and to follow the same mechanisms as in the gas dynamics case.

In the following section we will give numerical evidence of that. We translate two standard tests from Euler to shallow water and show how they produce the carbuncle. We also show that our simple carbuncle cure [10] has the same effect in shallow water as it has in gas dynamics. From this, we conclude that the same should be true for all other carbuncle cures which were developed for the Euler equations.

2. Numerical Investigation

In this section, we provide a numerical investigation of the carbuncle in shallow water and the effect of a carbuncle cure originally developed for gas dynamics. For the sake of simplicity, we resort to our own simple carbuncle cure, the HLLMCC-solver [10], which is designed to prevent the carbuncle while, like Roe, yielding exact resolution of steady shear waves. This can be literally translated to shallow water. The parameters given in [10, Section 4] are used with one difference: we can use $\varepsilon = 10^{-5}$ instead of $\varepsilon = 10^{-2}$. We compare this to the Roe and to the HLL solver. The computational domain for all test cases is $[-2.5, 2.5]$ discretized with 40×40 grid cells. The gravitational constant is normalized to one. The computations are done in clawpack [14] with first order and flat bed.

2.1. Colliding flow test

LeVeque [15, Section 7.7] outlined a test problem for the carbuncle instability. It consists of a pair of slowly moving shocks, initialised by a strong colliding flow. To trigger the carbuncle, LeVeque disturbs the initial state in one grid point. In this study, we simulate this situation for shallow water by setting the initial water-height to $h = 1$ and the left and right velocities to $u = \pm 30$. The transverse velocity is zero. Onto this initial state, we superimpose artificial numerical noise of amplitude 10^{-6} instead of disturbing it in just one point. The noise is generated randomly. This allows us to make sure that the resulting structure of the solution is independent of the structure of the initial perturbation. Figure 1 shows that, while the results for HLL and HLLMCC are indistinguishable, the Roe scheme produces carbuncle type structures.

2.2. Steady shock test

The steady shock problem was introduced by Dumbser et al. [9] as a test for the carbuncle. We consider the worst case: The shock is located directly on a cell face.

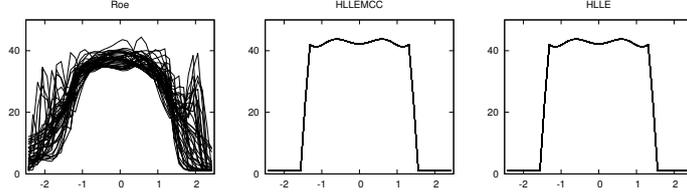


Figure 1: Scatter plot of colliding flow at time $t = 2$ with Roe, HLEMCC, HLL (left to right), water height shown.

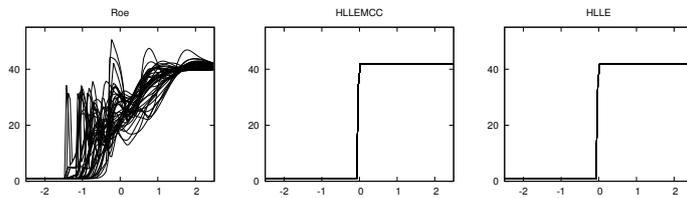


Figure 2: Scatter plot of steady shock with inflow Froude number $Fr = 30$ at time $t = 2$ with Roe, HLEMCC, HLL (left to right), water height shown.

According to Dumbser et al. [9] this situation is most likely to evolve a carbuncle-like structure. For shallow water we set the water height and the Froude number at the inflow to $h = 1$ and $Fr = 30$. Again, we add artificial numerical noise, this time of amplitude 10^{-3} , to the conserved variables in the initial state. Figure 2 shows results for the water height at time $t = 2$. Again, the Roe scheme shows a breakdown of the shock profile, while HLL and HLEMCC nicely reproduce the shock.

To illustrate the aforementioned interplay between 1d-instabilities of the shock and shear flows and vortices along the shock front, in Figure 3, we plot the transverse velocity. It is nicely seen that the Roe scheme leaves the shear flow along the shock undamped and, thus, let it grow until it starts to interact with the shock front and destroy it. Since along the strong shock the residual in the Rankine Hugoniot condition for the shear wave is rather large, HLEMCC puts almost the same amount of viscosity on it as with the HLL. The shear flow is damped and, thus, the resulting vortices are too weak to act back on the shock profile.

3. Conclusions and Outlook

We have given numerical evidence that the carbuncle in shallow water not only exists, but has even the same nature as in the gas dynamics case. It is driven by a complex interplay between one-dimensional shock instabilities and undamped unphysical vortices along the shock front. And it can be cured by a modified diffusion mechanism which was originally developed for the Euler equations. Thus, we expect

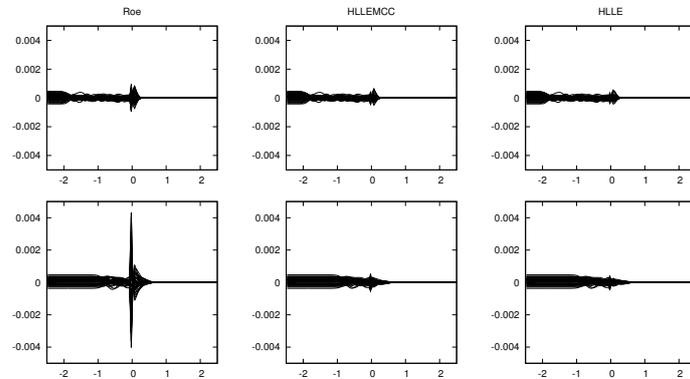


Figure 3: Scatter plot of transverse velocity component (v) at times $t = 0.002$ (upper row) and $t = 0.006$ (lower row) with Roe, HLEMCC, HLLC (left to right).

that any carbuncle cure developed for the Euler equations should also work for shallow water flows.

- [1] J. J. Quirk, A contribution to the great Riemann solver debate., *Int. J. Numer. Methods Fluids* 18 (6) (1994) 555–574. doi:10.1002/fld.1650180603.
- [2] R. Sanders, E. Morano, M.-C. Druguet, Multidimensional dissipation for upwind schemes: Stability and applications to gas dynamics, *J. Comput. Phys.* 145 (2) (1998) 511–537.
- [3] M. Bultelle, M. Grassin, D. Serre, Unstable Godunov discrete profiles for steady waves, *SIAM J. Numer. Anal.* 35 (6) (1998) 2272–2297.
- [4] M. Arora, P. L. Roe, On postshock oscillations due to shock capturing schemes in unsteady flows, *J. Comput. Phys.* 130 (1997) 25–40.
- [5] Y. Chauvat, J.-M. Moschetta, J. Gressier, Shock wave numerical structure and the carbuncle phenomenon, *Int. J. Numer. Methods Fluids* 47 (2005) 903–909.
- [6] S.-S. Kim, C. Kim, O.-H. Rho, S. K. Hong, Cures for the shock instability: Development of a shock-stable roe scheme, *J. Comput. Phys.* 185 (2003) 342–374.
- [7] M. Pandolfi, D. D. Ambrosio, Numerical instabilities in upwind methods: Analysis and cures for the carbuncle phenomenon, *J. Comput. Phys.* 166 (2) (2001) 271–301.
- [8] S. H. Park, J. H. Kwon, On the dissipation mechanism of Godunov type schemes, *J. Comput. Phys.* 188 (2003) 524–542.
- [9] M. Dumbser, J.-M. Moschetta, J. Gressier, A matrix stability analysis of the carbuncle phenomenon., *J. Comput. Phys.* 197 (2) (2004) 647–670. doi:10.1016/j.jcp.2003.12.013.
- [10] F. Kemm, A carbuncle free Roe-type solver for the Euler equations., Benzoni-Gavage, Sylvie (ed.) et al., *Hyperbolic problems. Theory, numerics and applications. Proceedings of the 11th international conference on hyperbolic problems, Ecole Normale Supérieure, Lyon, France, July 17–21, 2006.* Berlin: Springer. 601–608 (2008). (2008).
- [11] E. F. Toro, *Riemann solvers and numerical methods for fluid dynamics. A practical introduction.* 3rd ed., Berlin: Springer, 2009. doi:10.1007/b79761.
- [12] E. F. Toro, *Shock-capturing methods for free-surface shallow flows.*, Chichester: Wiley, 2001.
- [13] R. Denlinger, D. O’Connell, Closure to “Computing Nonhydrostatic Shallow-Water Flow over Steep Terrain” by Roger P. Denlinger and Daniel RH O’Connell, *Journal of Hydraulic Engineering* 137 (2011) 141–142.
- [14] Clawpack (conservation laws package), <http://www.amath.washington.edu/~claw>.
- [15] R. J. LeVeque, Nonlinear conservation laws and finite volume methods, in: O. Steiner, A. Gautschy

(Eds.), *Computational Methods for Astrophysical Fluid Flow*, Springer, Berlin, Heidelberg, 1998, pp. 1–160.