ISTT1004 UKE 6 * NORMALFORDELINGEN *

OPPG. 1

$X_L \sim \text{Poisson}(\lambda_L t)$ $X_G \sim \text{Poisson}(\lambda_G t)$

$\lambda_L = 0.5/\text{år}$ $t = 2$ år $\lambda_G = 2/\text{år}$ $t = 2$ år

Kostnader som stok. variabel

$$Y = 3000 \cdot X_L + 1000 \cdot X_G$$

$E(Y) = 7000$ kr $SD(Y) = 3605.551$ kr

NB: Vi kjenner ikke fordelingen til Y!

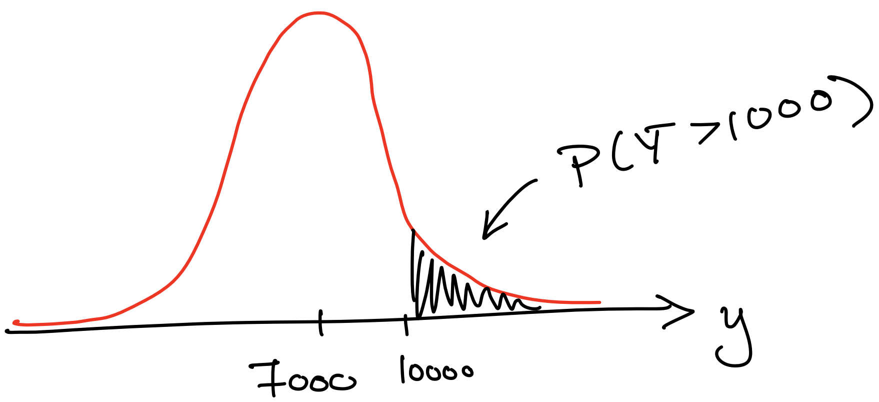
→ Stokastisk simulering

Vi skal altså ikke bruke den simulerte fordelingen, men approksimere med normal-fordeling.

Normaltilnærming

$$Y \sim N(\mu = 7000, \sigma = 3605.551)$$

$$P(Y > 1000) = ?$$



$$P(Y > 10000) = 1 - P(Y \leq 10000) = 1 - 0.7973 = 0.2027$$

$$\underline{\underline{\approx 0.20}}$$

Med tabell

$$\boxed{\frac{Y-\mu}{\sigma} \sim N(0,1)}$$

$\boxed{Z}$

Veldig viktig – og nyttig ! – egenskap med normalfordeling

$$P(Y \le 10000) = P\Big(\underbrace{\frac{Y-7000}{3605.551}}_{Z} \le \frac{10000-7000}{3605.551}\Big)$$

pga avrunding høyre side

$$\approx P(Z \le 0.83) = 0.7967$$

merk litt ulikt Python pga avrunding

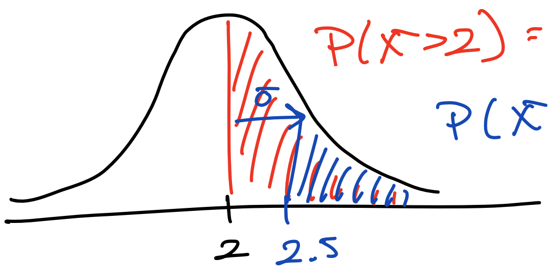
$$P(Y > 10000) = 1 - 0.7967 \approx 0.20$$

OPPG. 2

X : vekt av tilfeldig valgt hodehål

$X \sim N(\mu = 2,\ \sigma = 0.5)$

a) $P(2 \le X \le 2.5) = ?$



$P(X > 2) = 0.5$

$P(X > 2.5) \approx 0.16$

PGA ett standardavvik

$P(Z \ge 1)$ el

$P(Z \le -1)$

(se tabell / Regneøvelse mandag)

areal mellom 2 og 2.5 det vi er interessert i

$$\approx 0.5 - 0.16 = 0.34$$

Alternativt $P(2 \le X \le 2.5) = F(2.5) - F(2) =$

standard normal $\rightarrow$ $\Phi(1) - \Phi(0) = 0.8413 - 0.5000 \approx 0.34$

b) $\bar{X}_1$ = vekt hodehål 1

$\bar{X}_2$ = vekt hodehål 2 , anta $\bar{X}_1$ og $\bar{X}_2$ uavhengige

$$P(|\bar{X}_1 - \bar{X}_2| \leq 1) = ?$$

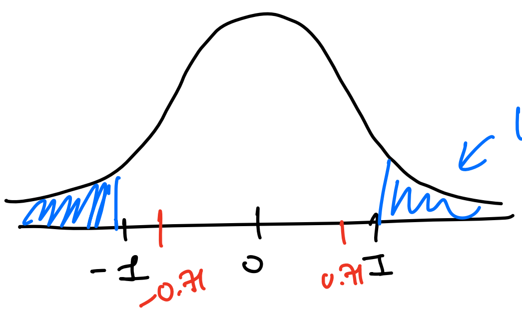
$$= P(-1 \leq \underbrace{\bar{X}_1 - \bar{X}_2}_{\text{hvilken fordeling har } \bar{X}_1 - \bar{X}_2\text{?}} \leq 1)$$

$\bar{X}_1 - \bar{X}_2$ er normalfordelt

$$E(\bar{X}_1 - \bar{X}_2) = 2 - 2 = 0$$

$$\mathrm{Var}(\bar{X}_1 - \bar{X}_2) = \mathrm{Var}(\bar{X}_1) \underset{\text{NB}}{+} \mathrm{Var}(\bar{X}_2) = 0{,}5^2 + 0{,}5^2$$

$$SD(\bar{X}_1 - \bar{X}_2) = \sqrt{2 \cdot 0{,}5^2} \approx 0.71$$



like store blå områder i hver hale

$$P(\bar{X}_1 - \bar{X}_2 \leq -1) = P\left(\frac{\bar{X}_1 - \bar{X}_2}{0.71} \leq \frac{-1}{0.71}\right) \approx P(Z \leq -1.41)$$

$$\overset{\text{tabell}}{=} 0.0793$$

$$P(-1 \leq \bar{X}_1 - \bar{X}_2 \leq 1) = 1 - \quad - $$

$$= 1 - 0.0793 - 0.0793$$

$$= 0.8414$$

c) $P(\bar{X} \geq k) = 0.05$

hva er k?

Tabell for Z:

$$P(Z > Z_\alpha) = \alpha$$

α ← 0.05, Z_α ← 1.645

$$P(\bar{X} \geq k) = P\left(\frac{\bar{X}-\mu}{\sigma} > \frac{k-\mu}{\sigma}\right) = 0.05$$

$$= P\Big(Z > \underbrace{\frac{k-2}{0.5}}_{1.645}\Big) = 0.05$$

$$\frac{k-2}{0.5} = 1.645 \qquad k = \underset{\mu}{2} + \underset{\sigma}{0.5} \cdot \underset{Z_\alpha}{1.645}$$

$$= 2.8225$$

$$\approx \underline{\underline{2.8 \text{ kg}}}$$

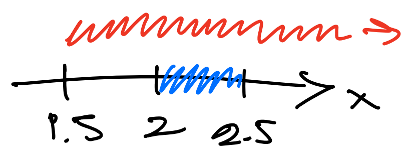
minst 2.8kg veier de 5% tyngste.

d) (hvis tid)

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(2 \leq \bar{X} \leq 2.5 \mid \bar{X} > 1.5)$$

$$= \frac{P(2 \leq \bar{X} \leq 2.5,\ \bar{X} > 1.5)}{P(\bar{X} > 1.5)}$$



snitt = blå

$$= \frac{P(2 \leq \bar{X} \leq 2.5)}{P(\bar{X} > 1.5)}$$ ← kjent

$$P(X>1.5) = 1-P(X \le 1.5) \approx 1-0.16 = 0.84$$

$(P(Z \le -1))$

⋮

$$= \frac{0.34}{0.84} \approx 0{,}40$$

Ja, logisk at sannsyn. for $2 \le X \le 2.5$ økte når vi begrenset utfallsrommet

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OPPG. 3

a) $$\bar{X} = \frac{\sum X_i}{n}$$

$$E(\bar{X}) = \frac{1}{n} E\left(\sum X_i\right) = \frac{1}{n} \sum E(X_i) = \frac{1}{n} n\mu = \underline{\underline{\mu}}$$

b) $$Var(\bar{X}) = \left(\frac{1}{n}\right)^2 Var\left(\sum X_i\right) \overset{\text{uavh.}}{=} \left(\frac{1}{n^2}\right) \sum Var(X_i)$$

$$= \left(\frac{1}{n}\right)^2 n\sigma^2 = \frac{\sigma^2}{n}$$

snittet har lavere 'usikkerhet' enn enkeltobservasjoner

$$\rightarrow SD(\bar{X}) = \sqrt{\frac{\sigma^2}{n}} = \frac{\sigma}{\sqrt{n}}$$

OPPG. 4

$X \sim N(100,\ 0.8)$ vekt av aluminiumsplate

a) V: Vekt av 10 plater + eske

$V = \underbrace{\sum_{i=1}^{10} X_i}_{\text{Vekt av } n=10 \text{ aluminiumsplater}} + 50$

$\boxed{\text{NB: } V \neq 10\bar{X} + 50}$

$E(V) = \sum_{i=1}^{10} E(X_i) + 50 = 10 \cdot 100 + 50$

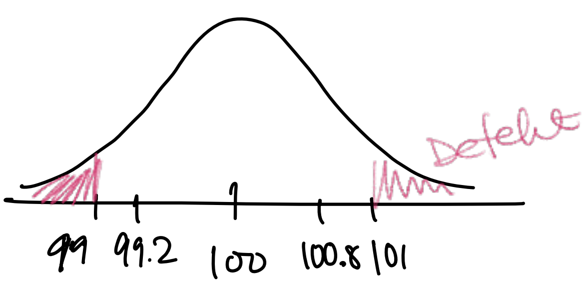
$= 1000 + 50 = 1050$ gram

b) $\underset{\text{uav}}{Var(V)} = \sum_{i=1}^{10} Var(X_i) + 0$ ← konstant har ikke varians

$= 10 \cdot 0.8^2$

$SD(V) = \sqrt{10 \cdot 0.8^2} \approx \underline{\underline{2.53}}$ g

c) $P(\text{defekt}) = \ ?$



$P(X \leq 99) =$

$P\left(Z \leq \frac{99-100}{0.8}\right) =$

$P(Z \leq -1.25) = 0.1056$

$P(\text{defekt}) = \ + \ = 2 \cdot 0.1056 \approx 0.21$

10 uavhengige forsøk $P(\text{"suksess"}) = 0.21 = p$

$Y \sim \text{binom}(n=10, p=0.21)$

ikke tabell for $p=0.21$

$P(Y \geq 2) = 1 - P(Y \leq 1)$

$= 1 - (P(Y=0) + P(Y=1))$

$\approx \underline{\underline{0.65}}$