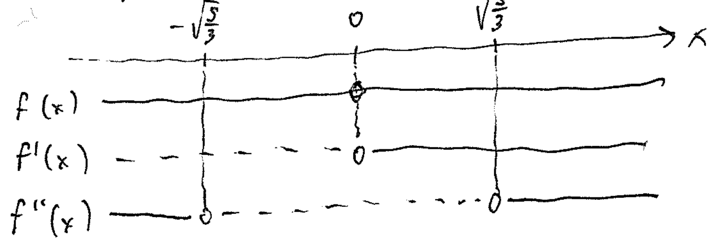
MA0001 2005H. Løsningsskisse

1a. $f'(x) = \dfrac{6x(x^2+5) - 3x^2 \cdot 2x}{(x^2+5)^2} = \dfrac{30x}{(x^2+5)^2}$.

$$f''(x) = \frac{30(x^2+5)^2 - 30x \cdot 2(x^2+5)\cdot 2x}{(x^2+5)^4} = \frac{30\left((x^2+5) - 4x^2\right)}{(x^2+5)^3} = \frac{-30(3x^2-5)}{(x^2+5)^3}$$

$$= \frac{-90\left(x - \sqrt{\tfrac{5}{3}}\right)\left(x + \sqrt{\tfrac{5}{3}}\right)}{(x^2+5)^3}$$

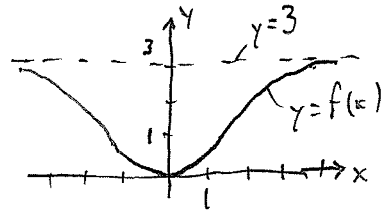
Fortegnsskjemaer for f, f' og f'':



(alle nevnere er positive)

f har nullpunkt i 0, (absolutt) minimum i 0 og vendepunkter i $-\sqrt{\frac{5}{3}}$ og $\sqrt{\frac{5}{3}}$.

b.



$\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} \dfrac{3}{1+5/x^2} = 3$, så $y=3$ er en horisontal asymptote.

c. Arealet er $\displaystyle\int_{-\infty}^{\infty}(3-f(x))\,dx = \int_{-\infty}^{\infty}\frac{3x^2+15-3x^2}{x^2+5}\,dx = 15\int_{-\infty}^{\infty}\frac{dx}{x^2+5}$.

Her er $x^2+5 = 5(x^2/5+1)$. La $u = \frac{x}{\sqrt{5}}$, $dx = \sqrt{5}\,du$.

$\displaystyle 15\int\frac{dx}{x^2+5} = 15\int\frac{\sqrt{5}\,du}{5(u^2+1)} = 3\sqrt{5}\arctan u + C$, dvs.

$$15\int_{-\infty}^{\infty}\frac{dx}{x^2+5} = \lim_{t\to\infty} 15\int_{-t}^{t}\frac{dx}{x^2+5} = \lim_{t\to\infty}\left[3\sqrt{5}\arctan\frac{x}{\sqrt{5}}\right]_{-t}^{t} = 3\sqrt{5}\cdot\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = 3\pi\sqrt{5}.$$

2. La $f(x) = e^{-x^2}$. $f'(x) = -2xe^{-x^2}$, $f''(x) = -2e^{-x^2} - 2xe^{-x^2}\cdot(-2x) = 2(2x^2-1)e^{-x^2}$.

$f(0)=1$, $f'(0)=0$, $f''(0)=-2$. Andregrads taylorpol. $1 + 0x - \frac{2}{2}x^2 = 1 - x^2$.

$\displaystyle\int_0^{1/2} e^{-x^2}dx \approx \int_0^{1/2}(1-x^2)\,dx = \left[x - \tfrac{1}{3}x^3\right]_0^{1/2} = \frac{1}{2} - \frac{1}{3}\cdot\frac{1}{8} = \frac{11}{24} \approx 0{,}45833\ldots$ (sann verdi $\approx 0{,}4613$).

3a. Impl. deriv.: $3x^2 + 3y^2\dfrac{dy}{dx} = 9y + 9x\dfrac{dy}{dx}$, $(3y^2-9x)\dfrac{dy}{dx} = 9y - 3x^2$, $\dfrac{dy}{dx} = \dfrac{3y-x^2}{y^2-3x}$

I $(2,4)$: $\dfrac{dy}{dx} = \dfrac{3\cdot 4 - 4}{16 - 3\cdot 2} = \dfrac{8}{10} = \dfrac{4}{5}$.

b. $y - 4 = \frac{4}{5}(x-2)$, $4x - 5y + 12 = 0$, evt. $y = \frac{4}{5}x + \frac{12}{5}$.