

Contact:
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English

FINAL EXAM, MA0002

Tuesday May 22. 2007
Time: 15.00 – 19.00
Grades: June 12. 2007

Permitted aids:
All printed and handwritten, calculator.

Justify all answers!
Show calculations, or refer to theory.

Problem 1 Matrixes:

- a) Find the two eigenvalues of the matrix $A = \begin{bmatrix} 4 & -2 \\ 5 & -3 \end{bmatrix}$, and find the corresponding eigenvectors.
- b) Calculate $A^{20} \begin{bmatrix} 3 \\ 6 \end{bmatrix}$.

Problem 2 Differential equations:

- a) Solve the initial value problem $\frac{dy}{dx} = x^2$, when $y(3) = 5$.
- b) Find all equilibria of the equation in a). Are the equilibria you found stable or unstable?
- c) Solve the initial value problem $t \frac{dx}{dt} = 2x + t^3 e^t$, when $x(0) = -1$.

Problem 3 In this problem we are studying this equation:

$$f(x, y) = 2(x + y^2) - x^3$$

- a) Find ∇f , the gradient of f .
- b) Find the critical points of f .
- c) Given the parametrized line $l(t)$

$$\begin{aligned}x &= 2t \\ y &= -3t + 1,\end{aligned}$$

where $-3 \leq t \leq 3$. This means that the line starts in the point $(-6, 10)$ and ends in the point $(6, -8)$. What is the absolute max and what is the absolute min f achieves on the points of this line?

- d) What is the directional derivative of f along the vector $\underline{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, in the point where the line from c) intersects the line given by $x = y$?
- e) Find an equation for the tangent plane of f in the point $(1, 1)$.