Norwegian University of Science
and Technology
Department of Mathematical
Sciences

MA0002 Mathematical
Methods B
Spring 2023

Exercise set 12

$\boxed{1}$ Let $f(x_1, x_2) = x_1 e^{x_2}$, where $x_1(t) = e^t$ and $x_2(t) = t^2$. Find the derivative of $w = f(x, y)$ with respect to t when $t = 0$, that is find $\frac{df}{dt}\big|_{t=0}$.

$\boxed{2}$ Let $f(x_1, x_2) = \ln(x_1 x_2 - x_1^2)$ where $x_1(t) = t^2$ and $x_2(t) = t$. Find the derivative of $w = f(x_1, x_2)$ with respect to t when $t = 5$, that is find $\frac{df}{dt}\big|_{t=5}$.

$\boxed{3}$ Find the gradient of the following functions.

(a)
$$f(x_1, x_2) = x_1(x_1^2 - x_2^2)^{2/3}$$

(b)
$$g(x_1, x_2) = \sin(3x_1^2 - 2x_2)$$

$\boxed{4}$ Let $f(x_1, x_2) = x_1^3 x_2^2$. Find the directional derivative of $f(x_1, x_2)$ in the point $(2, 3)$ and the direction $(-2, 1)^T$.

$\boxed{5}$ Let $f(x_1, x_2) = e^{x_1} \cos x_2$. In which direction does the function $f(x_1, x_2)$ increase the most in the point $(0, \pi/2)$?

$\boxed{6}$ Given the following function; find all the critical points and classify them.

(a)
$$f(x_1, x_2) = -2x_1^2 - x_2^2 + 3x_2$$

(b)
$$f(x_1, x_2) = x_1^3 - 2x_2^2 + 2x_1 x_2$$