

- 1 Find the global maxima and minima for the function

$$f(x_1, x_2) = x_1^2 + x_2^2 - 6x_2 + 3$$

on the disk

$$D = \{(x_1, x_2) : x_1^2 + x_2^2 \leq 16\}.$$

- 2 A rectangular prism consists of six rectangles. Find the largest possible volume of a rectangular prism when the surface area is 200m^2 .

- 3 Use Lagrange multipliers to find the maxima and minima of the functions under the given constraints.

(a) $f(x_1, x_2) = x_1 \cdot x_2$ when $x^2 + y^2 = 4$

(b) $f(x_1, x_2) = x_1^2 - x_2^2$ when $2x_1 + x_2 = 1$

- 4 Solve the following initial-value problem

$$\begin{pmatrix} \frac{dy_1}{dt} \\ \frac{dy_2}{dt} \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix},$$

where $y_1(0) = 2$ and $y_2(0) = -1$.

- 5 Solve the following initial-value problem

$$\begin{pmatrix} \frac{dy_1}{dt} \\ \frac{dy_2}{dt} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix},$$

where $y_1(0) = -1$ and $y_2(0) = -2$.