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MA0002 Brukerkurs i Matematikk B
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Exercise set 14

1 In this exercise we are working with the matrices

$$A = \begin{pmatrix} 0 & 1 & 2 \\ -1 & 3 & 2 \\ 1 & -4 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2 & -7 & 2 \\ 1 & 4 & 3 \\ 0 & 0 & -1 \end{pmatrix}.$$

(a) Calculate AB

(b) Is $AB = BA$? Justify your answer.

(c) Determine A^{-1}.

(d) Solve the equation $A\mathbf{x} = (1,1,1)^T$.

2 Let $f(x_1, x_2) = \cos(x_1^2 - 2x_2) + e^{x_1} + x_2^3$. Find the gradient of f.

3 Let

$$L = \begin{bmatrix} 0 & 5 \\ 0.9 & 0 \end{bmatrix}$$

be the Leslie matrix for a population consisting of two age groups.

a) Find both the eigenvalues for the matrix L.

b) Give a biological interpretation of the largest eigenvalue.

c) Find the stable age distribution.

4 Find the global maximum and minimum of $f(x_1, x_2) = x_1^2 + x_2^2 + x_1 + 2x_2$ if they exist.

5 Solve the following initial-value problem.

$$\begin{pmatrix} \frac{dy_1}{dt} \\ \frac{dy_2}{dt} \end{pmatrix} = \begin{pmatrix} -3 & 4 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix},$$

where $y_1(0) = 1$ and $y_2(0) = 2$.