MA0002 Brukerkurs i Matematikk B
Vår 2023

Exercise set 4

1 Let $N(t)$ be the size of a population after t years. Assume that

$$\frac{dN}{dt} = 0.7N\left(1 - \frac{N}{35}\right),$$

and that $N(0) = 10$. Solve the differential equation, and find $\lim_{t\to\infty} N(t)$. What does the limit value tell you?

2 Let

$$\frac{dy}{dx} = (4-y)(5-y).$$

Find the point equilibria and determine the stability of the equilibria.

3 Assume that $N(t)$ is the size of a population at time t and that N satisfies the differential equation

$$\frac{dN}{dt} = N\left(1 - \frac{N}{50}\right) - \frac{9N}{5+N}.$$

Let

$$g(N) = N\left(1 - \frac{N}{50}\right) - \frac{9N}{5+N}$$

(a) Sketch the graph of g.
(b) Find the equilibria of the differential equation.
(c) Determine the stability of the equilibria.

4 Assume the single-compartment model defined in subsection 8.2.2. That is assume that $C(t)$ is the concentration of the solute at time t and assume that

$$\frac{dC}{dt} = 3(20 - C(t)).$$

(a) Solve the equation above when $C(0) = 5$.
(b) Find $\lim_{t\to\infty} C(t)$.
(c) Use your answer from (a) to find t when $C(t) = 10$.

5 Let $N(t)$ be the size of a population at the time t. Assume that

$$\frac{dN}{dt} = 2N(N-10)\left(1 - \frac{N}{100}\right),$$

for $t \geq 0$.

(a) Find all the equilibria for this differential equation.

(b) Determine the stability of the equilibrium.

6 Assume the classical Kermack–McKendrick model for spread of disease (see chapter 8.3.1 in the book). Decide whether the disease in the 2 cases below will spread or not by determening if R_0 is greater than, or less than 1.

(a) $S(0) = 1000$, $b = 0.4$, $a = 300$

(b) $S(0) = 500$, $b = 0.1$, $a = 200$.