

1 In this exercise we look at what a Leslie matrix tells us about. Let

$$L = \begin{pmatrix} 0 & 5 & 2 \\ 0.8 & 0 & 0 \\ 0 & 0.3 & 0 \end{pmatrix}$$

be a Leslie matrix describing a population.

- (a) Determine the average number of female offsprings of the two-year-olds.
- (b) Determine the average number of female offsprings of the one-year-olds.
- (b) What can you say about the rate of survival of zero-year-olds and one-year-olds in this population?

2 Assume that we have a population consisting of zero-year-olds, one-year-olds and two-year-olds. Assume that 80% of the female zero-year-olds and 10% of the female one-year-olds survive until the next breeding season. Assume further that female one-year-olds have an average of 1.6 female offspring and that female two-year-olds have an average of 3.9 female offspring. We assume that at time $t = 0$ the population consists of 1000 female zero-year-olds, 100 female one-year-olds and 20 female two-year-olds.

- (a) Find the Leslie matrix L that describes the situation above.
- (b) Find the age distribution at time $t = 2$. That means; how many female zero-year-olds, one-year-olds and two-year-olds does the population consist of after two years?

3 In this exercise we will consider the vectors

$$x = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad y = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad \text{og} \quad z = \begin{pmatrix} 2 \\ 2 \end{pmatrix}.$$

- (a) Calculate $2x - y$ and illustrate the calculation with a drawing in the plane.
- (b) Find the polar coordinates of the vector z and draw the vector.

4 Given the following vectors in polar coordinates (r, θ) , find the cartesian coordinates of the vector.

- (a) $r = 4, \theta = \frac{\pi}{2}$

(b) $r = 2, \theta = \frac{4\pi}{3}$

5 For each of the maps, determine whether they are linear or not.

(a)

$$T\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right) = \begin{pmatrix} 2x_1 \\ 3x_2 \end{pmatrix}$$

(b)

$$T\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right) = \begin{pmatrix} 3x_1 + 2 \\ x_2 \end{pmatrix}$$

6 Let

$$A = \begin{pmatrix} 3 & 6 \\ 1 & 4 \end{pmatrix} \quad \text{and,} \quad B = \begin{pmatrix} -1 & 4 \\ 0 & -2 \end{pmatrix}.$$

For each of the matrices A and B , find the eigenvalues and the corresponding eigenvectors.