Norwegian University of Science and Technology
Department of Mathematical Sciences

MA0002 Mathematical Methods B
Spring 2024

Exercise set 6

1 Let

$$A = \begin{pmatrix} 2 & -1 & 9 \\ 0 & 3 & -1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 9 & -7 & 3 \\ -1 & -2 & 6 \end{pmatrix}.$$

(a) Calculate $A + B$.

(c) Find the matrix C such that $A + B + C = \mathbf{0}$.

2 Let

$$A = \begin{pmatrix} 3 & 0 & 1 \\ -1 & -2 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 1 \\ 7 & -1 \\ 0 & -2. \end{pmatrix}$$

(a) Calculate AB.

(b) Calculate BA.

3 Let

$$A = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$$

(a) Calculate $3A$.

(b) Calculate A^3.

(c) What is the inverse matrix of A?

4 Let

$$A = \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \quad \text{and} \quad C = \begin{pmatrix} 1 & 3 & 0 \\ 0 & -1 & 2 \\ -1 & -2 & 1 \end{pmatrix}.$$

For each of the matrices A, B and C; find the inverse matrix or explain why it does not exist.

5 Let

$$A = \begin{pmatrix} 1 & 2 \\ -1 & a \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

(a) Show that for $a \neq -2$ the equation $A\mathbf{x} = \mathbf{b}$ has exactly one solution.

(b) Let $a = 1$. Solve the equation

$$A\mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

by using the inverse matrix A^{-1}.

(c) Let $a = -2$. Determine conditions on b_1 and b_2 such that the equations $A\mathbf{x} = \mathbf{b}$ has

(i) infinitely many solutions,

(ii) no solutions.

(d) Explain your results from (a), (b) and (c) graphically. Tip: think about what the slope of the equations tells you.