



1 Use the properties for limits to compute the following limits

a)

$$\lim_{(x_1, x_2) \rightarrow (-1, 1)} (2x_1x_2 + 3x_1^2)$$

b)

$$\lim_{(x_1, x_2) \rightarrow (1, 1)} \frac{x_1x_2}{x_1^2 + x_2^2}$$

2 Show that

$$\lim_{(x_1, x_2) \rightarrow (0, 0)} \frac{3x_1^2 - x_2^2}{x_1^2 + x_2^2}$$

does not exist by computing the limit along the positive x_1 -axis and the positive x_2 -axis.

3 a) Compute

$$\lim_{(x_1, x_2) \rightarrow (0, 0)} \frac{3x_1x_2}{x_1^2 + x_2^2}$$

along the lines $x_2 = mx_1$, for $m \neq 0$. What can you conclude?

b) Show that

$$f(x_1, x_2) = \begin{cases} \frac{3x_1x_2}{x_1^2 + x_2^2} & \text{for } (x_1, x_2) \neq (0, 0) \\ 0 & \text{for } (x_1, x_2) = (0, 0) \end{cases}$$

is discontinuous at $(0, 0)$.

4 Find $\partial f / \partial x_1$ and $\partial f / \partial x_2$ for the functions

a) $f(x_1, x_2) = 2x_1\sqrt{x_2} - \frac{3}{x_1x_2^2}$

b) $f(x_1, x_2) = e^{-x_1^2} \cos(x_1^2 - x_2^2)$

5 Let

$$f(x_1, x_2) = 2x_1^3 - 3x_2x_1.$$

Compute $f_{x_1}(1, 2)$ and $f_{x_2}(1, 2)$, and give a representation of the tangent plane of f at $(1, 2)$.

6 Let

$$\mathbf{f}(x_1, x_2) = \begin{bmatrix} 3x_1 - x_2^2 \\ 4x_2 \end{bmatrix}.$$

Find the linear approximation of $\mathbf{f}$ at $(-1, -2)$.