Norwegian University of Science
and Technology
Department of Mathematical
Sciences

MA0002 Mathematical Methods B Spring 2024

Exercise set 3

1 Solve the pure-time differential equation:

$$\frac{dx}{dt} = \sin\left(2\pi(t+1)\right), \text{ where } x(2) = 1.$$

2 Suppose that the amount of phosphorus in a lake at time t, denoted by $P(t)$, follows the equation

$$\frac{dP}{dt} = 2t + 3 \quad \text{with } P(0) = 0.$$

Find the amount of phosphorus at time $t = 5$.

3 Solve the autonomous differential equations:

$$\frac{dx}{dt} = 2 - x, \text{ where } x(-2) = -1.$$

4 Assume that $W(t)$ denotes the amount of radioactive material in a substance at time t. Radioactive decay is then described by the differential equation

$$\frac{dW}{dt} = -\lambda W(t), \quad \text{with } W(0) = W_0.$$

(a) Find $W(t)$ by solving the differential equation.

(b) Assume that $W(0) = 123\text{gr}$, $W(5) = 20\text{gr}$ and that time is measured in minutes. Find the decay constant λ and determine the half-life of the radioactive substance.

5 Use the partial fraction decomposition to solve

$$\frac{dy}{dx} = y(y-1), \text{ where } y_0 = 1 \text{ if } x_0 = 0.$$

6 Solve the differential equation:

$$\frac{dy}{dx} = x^2 e^y, \text{ where } y_0 = 1 \text{ if } x_0 = 1.$$