



- 1 Let $N(t)$ be the size of a population after t years. Assume that

$$\frac{dN}{dt} = 0.8N \left(1 - \frac{N}{27}\right),$$

and that $N(0) = 2$. Solve the differential equation, and find $\lim_{t \rightarrow \infty} N(t)$. What does the limit value tell you?

- 2 Let

$$\frac{dy}{dx} = (2 - y)(3 - y).$$

Find the point equilibria and determine the stability of the equilibria.

- 3 Assume that $N(t)$ is the size of a population at time t and that N satisfies the differential equation

$$\frac{dN}{dt} = N \left(1 - \frac{N}{50}\right) - \frac{9N}{5 + N}.$$

Let

$$g(N) = N \left(1 - \frac{N}{50}\right) - \frac{9N}{5 + N}$$

- (a) Sketch the graph of g .
- (b) Find the equilibria of the differential equation.
- (c) Determine the stability of the equilibria.

- 4 Assume the single-compartment model defined in subsection 8.2.2. That is assume that $C(t)$ is the concentration of the solute at time t and assume that

$$\frac{dC}{dt} = 5(4 - C(t)).$$

- (a) Solve the equation above when $C(0) = 2$.
- (b) Find $\lim_{t \rightarrow \infty} C(t)$.
- (c) Use your answer from (a) to find t when $C(t) = 3$.

- 5 Let $N(t)$ be the size of a population at the time t . Assume that

$$\frac{dN}{dt} = 5N(N - 2) \left(1 - \frac{N}{10}\right),$$

for $t \geq 0$.

- (a) Find all the equilibria for this differential equation.
 - (b) Determine the stability of the equilibrium.
- 6 Assume the classical Kermack–McKendrick model for spread of disease (see chapter 8.3.1 in the book). Decide whether the disease in the 2 cases below will spread or not by determining if R_0 is greater than, or less than 1.
- (a) $S(0) = 1000$, $b = 0.4$, $a = 300$
 - (b) $S(0) = 500$, $b = 0.1$, $a = 200$.