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Trigonometriske funksjoner

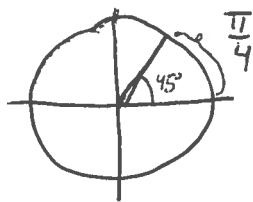
Husk: en sirkel med radius 1 har omkrets 2π .

Vinkler: kan måles i grader eller radianer

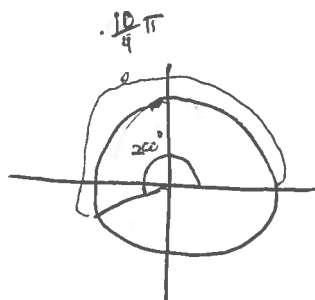
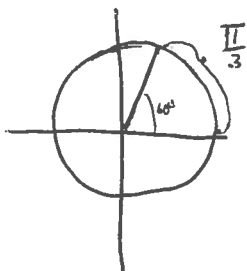
$$360^\circ = 2\pi \text{ radianer}$$

$$1^\circ = \frac{2\pi}{360} = \frac{\pi}{180} \text{ radianer}$$

eks 45° er $\frac{1}{8}$ av en hel sirkel, $\frac{1}{8} \cdot 2\pi = \frac{\pi}{4}$



60° er $\frac{1}{6}$ av en hel sirkel $\frac{1}{6} \cdot 2\pi = \frac{\pi}{3}$



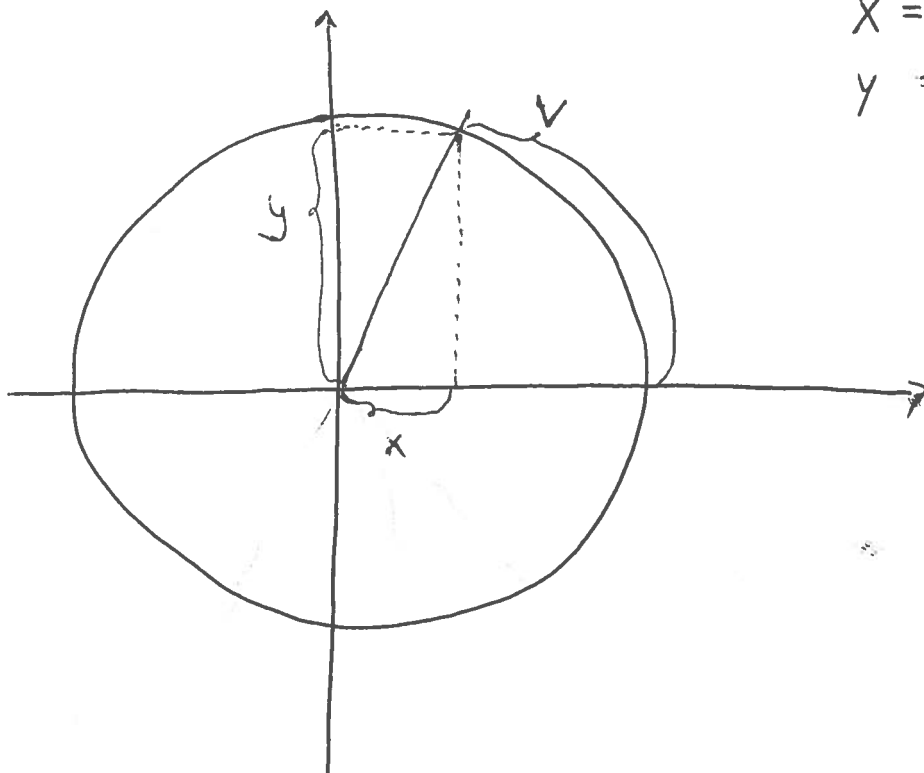
$$\begin{aligned} 200^\circ &= 200 \cdot \frac{\pi}{180} \text{ radianer} \\ &= \frac{10}{9} \pi \end{aligned}$$

Vil definere to funksjoner: sinus og cosinus.

Definerer de først på intervallet $[0, 2\pi) \in \mathbb{R}$.

Etterpå: utvider til hele \mathbb{R} .

La $v \in [0, 2\pi)$,



$$x = \cos v$$

$$y = \sin v$$

Ex

$$\sin 45^\circ = \cos 45^\circ$$

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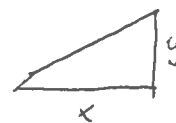
$$\sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\sin 135^\circ = \sin 45^\circ = \frac{1}{\sqrt{2}}$$

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$$\sin \frac{3\pi}{4} = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\cos \frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$$



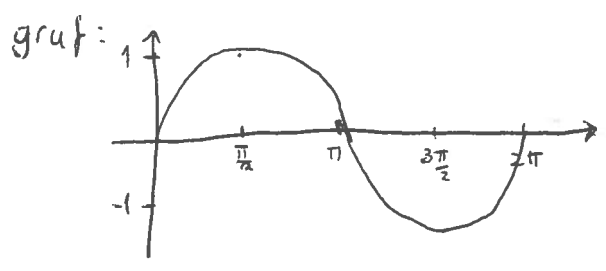
$$x = y \text{ og } x^2 + y^2 = 1$$

$$\Rightarrow x = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

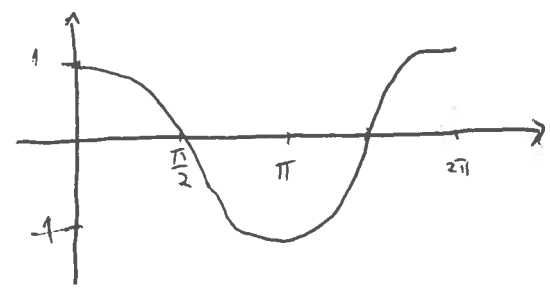
Ex. $\sin 0 = 0$
 $\cos 0 = 1$

$\sin \frac{\pi}{2} = 1$
 $\cos \frac{\pi}{2} = 0$

$y = f(x) = \sin x$
 $f: [0, 2\pi) \rightarrow [-1, 1] \subseteq \mathbb{R}$



$y = g(x) = \cos x$
 $g: [0, 2\pi) \rightarrow [-1, 1] \subseteq \mathbb{R}$



Vil atvide $f(x) = \sin x$, $g(x) = \cos x$ s.a. $D_f = D_g = \mathbb{R}$

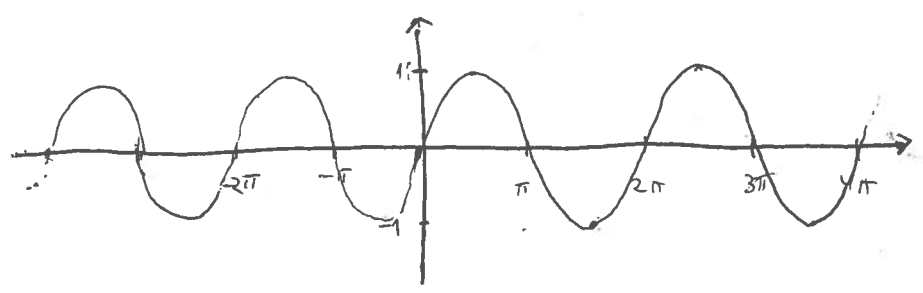
Definer $\sin(x + n \cdot 2\pi) = \sin x$ for alle $n \in \mathbb{Z}$

og $\cos(x + n \cdot 2\pi) = \cos x$ for alle $n \in \mathbb{Z}$

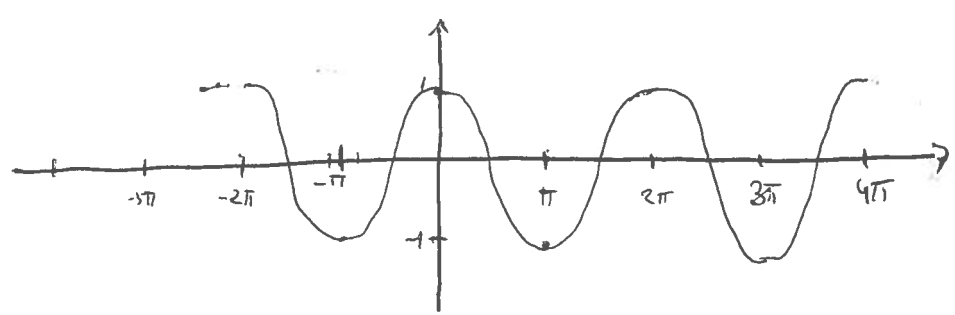
Har da defineret $f(x) = \sin x$ og $g(x) = \cos x$ for alle $x \in \mathbb{R}$

Gratene

$f(x) = \sin x$



$g(x) = \cos x$



Sinus og cosinus er eksempler på såkalt
periodiske funksjoner.

En funksjon $f: \mathbb{R} \rightarrow \mathbb{R}$ kalles periodisk ^{med periode a} hvis $f(x) = f(x+a)$ for alle $x \in \mathbb{R}$.

Sinus og cosinus er periodisk med periode $a = 2\pi$

Merk: $\sin(-x) = \sin(2\pi - x) = -\sin x$

$$\cos(-x) = \cos x$$

Merk: $\sin(x + \pi) = -\sin x$

$$\cos(x + \pi) = -\cos x$$

Merk: $\sin^2 x + \cos^2 x = 1$ (fra Pythagoras)

$$\frac{\sin^2 x + \cos^2 x}{(\sin x)^2 + (\cos x)^2} = 1$$

Tangens

$$h(x) = \tan x = \frac{\sin x}{\cos x}$$

$$D_h = \mathbb{R} \setminus \left\{ \frac{\pi}{2} + n\pi \mid n \in \mathbb{Z} \right\}$$

↑

dette er nullpunktene
til $\cos x$

Oppgave: $h(x)$ er periodisk med
periode π .