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Solutions to exercise set 6

1 Compute the following limits.

a) $\lim_{x\to 0} \cos x = \cos 0 = 1$

b) $\lim_{x\to 0} (x \sin x) = 0 \cdot \sin 0 = 0$

c) $\lim_{x\to \frac{\pi}{2}} \frac{\cos x}{\sin x} = \frac{\cos(\pi/2)}{\sin(\pi/2)} = \frac{0}{1} = 0$

d) $\lim_{x\to \frac{\pi}{3}} \frac{2x + \cos(x)}{x \sin(x)} = \frac{\frac{2\pi}{3} + \cos(\pi/3)}{\frac{\pi}{3}\sin(\pi/3)} = \frac{\frac{2\pi}{3} + \frac{1}{2}}{\frac{\sqrt{3}\pi}{6}}$

2 Differentiate the following functions.

a) $f(x) = x^{100} + x^{50} + \cos x \Rightarrow f'(x) = 100x^{99} + 50x^{49} - \sin x$

b) $f(x) = \sin(2x)\cos(3x) \Rightarrow f'(x) = 2\cos(2x)\cos(3x) - 3\sin(2x)\sin(3x)$

c) $f(x) = \cos(e^x) \Rightarrow -e^x \sin(e^x)$

d) $f(x) = \sin(\ln x) \Rightarrow f'(x) = \cos(\ln x) \cdot \frac{1}{x} = \frac{\cos(\ln x)}{x}$

e) $f(x) = (x-1)^3\cos(x^4) \Rightarrow f'(x) = 3(x-1)^2\cos(x^4) - (x-1)^3\sin(x^4)\cdot 4x^3$

3 Find the equation for the tangent line to the function $f(x) = \cos x$ at the point $(\pi/3, 1/2)$.

Solution: Recall that one way we can write the equation for a line (though not the only way) is given by $y - y_1 = m(x - x_1)$, where m is the slope, and (x_1, y_1) is a point on the line. The slope m is given by the derivative of f, which in this case is

$$f'(x) = -\sin x.$$

To determine the slope at the appropriate point, we compute

$$f'\left(\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}.$$

Based on the information given in the question, we use $(\pi/3, 1/2)$ for (x_1, y_1). Substituting all these things gives us

$$y - \frac{1}{2} = -\frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right).$$