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Solutions to exercise set 11

1.4:1 From the definition of matrix-vector multiplication on page 41, we see that the product

$$\begin{bmatrix} -4 & 2 \\ 1 & 6 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 7 \end{bmatrix}$$

is undefined, as the number of columns in the first matrix does not match the number of rows in the vector.

1.4:11 The augmented matrix corresponding to $A\mathbf{x} = \mathbf{b}$ is simply

$$\begin{bmatrix} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ -2 & -4 & -3 & -1 \end{bmatrix}.$$

To solve the system, we simply perform the following elementary row operations

$$\begin{bmatrix} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ -2 & -4 & -3 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 5 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -6 & -6 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 5 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

From the last matrix, we see that the solutions are $x_1 = 0$, $x_2 = -3$, and $x_3 = 1$.

1.8:2 From the definition of matrix multiplication, we have

$$\begin{aligned} \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -4 \end{bmatrix} &= 1 \begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix} + 0 \begin{bmatrix} 0 \\ 0.5 \\ 0 \end{bmatrix} - 4 \begin{bmatrix} 0 \\ 0 \\ 0.5 \end{bmatrix} \\ &= \begin{bmatrix} 0.5 \\ 0 \\ -2 \end{bmatrix}. \end{aligned}$$

Similarly, we can compute $A\mathbf{v}$ by

$$\begin{aligned} \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} &= a \begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 0.5 \\ 0 \end{bmatrix} + c \begin{bmatrix} 0 \\ 0 \\ 0.5 \end{bmatrix} \\ &= \begin{bmatrix} 0.5a \\ 0.5b \\ 0.5c \end{bmatrix}. \end{aligned}$$

1.8:9 To find all the vectors in $\mathbb{R}$ that are mapped to zero, we assume we have some vector $\mathbf{x}$ for which the product $A\mathbf{x} = \mathbf{0}$. As in exercise 11 from section 1.4, we need only write the augmented matrix

$$\begin{bmatrix} 1 & -4 & 7 & -5 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 2 & -6 & 6 & -4 & 0 \end{bmatrix}$$

and perform elementary row operations to row-reduce the matrix. To row-reduce, we perform the following elementary row operations:

$$\begin{bmatrix} 1 & -4 & 7 & -5 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 2 & -6 & 6 & -4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 7 & -5 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 2 & -8 & 6 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -9 & 7 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

To obtain our solutions from this, we note that this augmented matrix gives us infinitely many solutions of the form

$$\begin{aligned} x_1 &= -9x_3 + 7x_4 \\ x_2 &= -4x_3 + 3x_4, \end{aligned}$$

where x_3 and x_4 are arbitrary. Thus, any vector of the form

$$\begin{bmatrix} -9x_3 + 7x_4 \\ -4x_3 + 3x_4 \\ x_3 \\ x_4 \end{bmatrix}$$

is mapped to $\mathbf{0}$ by the matrix A.

1.9:1 Recall that the matrix of a linear transformation can be computed by determining what happens to a set of linearly independent vectors which spans $\mathbb{R}^2$. Since the given vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ span $\mathbb{R}^2$ (why?), then the resulting matrix is just

$$\begin{bmatrix} T(\mathbf{e}_1) & T(\mathbf{e}_2) \end{bmatrix}.$$

Thus

$$A = \begin{bmatrix} 3 & -5 \\ 1 & 2 \\ 3 & 0 \\ 1 & 0 \end{bmatrix}.$$