Norwegian University of Science and Technology
Institutt for matematiske fag

MA0003 Brukerkurs i matematikk for informatikere
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Solutions to exercise set 12

2.1:1 **a)** By definition, we have

$$\begin{aligned} -2A &= -2\begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -4 & 0 & 2 \\ -8 & 10 & -4 \end{bmatrix} \end{aligned}$$

b) As before,

$$\begin{aligned} B - 2A &= B + (-2A) \\ &= \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix} + \begin{bmatrix} -4 & 0 & 2 \\ -8 & 10 & -4 \end{bmatrix} \\ &= \begin{bmatrix} 3 & -5 & 3 \\ -7 & 6 & -7 \end{bmatrix} \end{aligned}$$

c) Undefined.

d) Using the definition on page 110, the product is given by

$$CD = \begin{bmatrix} C\mathbf{d}_1 & C\mathbf{d}_2 \end{bmatrix},$$

where $C\mathbf{d}_i$ is the ith column of the matrix D. Thus

$$\begin{aligned} CD &= \begin{bmatrix} C\mathbf{d}_1 & C\mathbf{d}_2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 13 \\ -7 & -6 \end{bmatrix} \end{aligned}$$

2.1:2 **a)** As before,

$$\begin{aligned} A + 2B &= \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix} + 2\begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix} \\ &= \begin{bmatrix} 16 & -10 & 1 \\ 6 & -13 & -4 \end{bmatrix} \end{aligned}$$

b) Undefined.

c) By definition,

$$\begin{aligned} CB &= \begin{bmatrix} C\mathbf{b}_1 & C\mathbf{b}_2 & C\mathbf{b}_3 \end{bmatrix} \\ &= \begin{bmatrix} 9 & -13 & -5 \\ -13 & 6 & -5 \end{bmatrix} \end{aligned}$$

d) Undefined.

2.2:1 Recall that the first step in finding the inverse of a matrix is to augment it with the identity matrix:

$$\begin{bmatrix} 8 & 6 & 1 & 0 \\ 5 & 4 & 0 & 1 \end{bmatrix}.$$

Then, we row-reduce:

$$\begin{bmatrix} 8 & 6 & 1 & 0 \\ 5 & 4 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 8 & 6 & 1 & 0 \\ 0 & 2 & -5 & 8 \end{bmatrix} \sim \begin{bmatrix} 8 & 0 & 16 & -24 \\ 0 & 2 & -5 & 8 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & 1 & -\frac{5}{2} & 4 \end{bmatrix}$$

We can now obtain our answer from the last matrix. It is:

$$\begin{bmatrix} 2 & -3 \\ -\frac{5}{2} & 4 \end{bmatrix}.$$

2.2:1 As before, the first step in finding the inverse of a matrix is to augment it with the identity matrix:

$$\begin{bmatrix} 3 & 2 & 1 & 0 \\ 7 & 4 & 0 & 1 \end{bmatrix}.$$

Then, we row-reduce:

$$\begin{bmatrix} 3 & 2 & 1 & 0 \\ 7 & 4 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 3 & 2 & 1 & 0 \\ 0 & -2 & -7 & 3 \end{bmatrix} \sim \begin{bmatrix} 3 & 0 & -6 & 3 \\ 0 & -2 & -7 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & \frac{7}{2} & -\frac{3}{2} \end{bmatrix}$$

We can now obtain our answer from the last matrix. It is:

$$\begin{bmatrix} -2 & 1 \\ \frac{7}{2} & -\frac{3}{2} \end{bmatrix}.$$

2.7:2 Recall that, if D is the data matrix and A is the matrix that corresponds to a geometric transformation, then we can represent the result after the transformation by the matrix product AD. Since we are working with a a reflection (and not a translation) of coordinates, we don't need to use homogeneous coordinates. Thus, we need only compute AD, where D is the given matrix (unmodified), and A is the transformation with matrix

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}.$$

(See page 85 in the textbook.) Computing this product gives us

$$AD = \begin{bmatrix} -5 & -2 & -4 \\ 0 & 2 & 3 \end{bmatrix}.$$