

**MA0301 ELEMENTARY DISCRETE MATHEMATICS  
NTNU, SPRING 2018**

1. HOMEWORK SET 12

**Exercise 1.** (Grimaldi, 5. ed., Exercises 6.1, page 317) Exercise 18 (Proof of Thm. 6.2 in  $G$ 's book, i.e., Thms. IX. 8 of the course)

**Exercise 2.** (Grimaldi, 5. ed., Exercises 6.1, page 317) Exercise 1

**Exercise 3.** (Grimaldi, 5. ed., Exercises 6.1, page 317) Exercise 9

**Exercise 4.** (Grimaldi, 5. ed., Exercises 6.1, page 317) Exercise 12 d), e), f)

**Exercise 5.** (Grimaldi, 5. ed., Exercises 6.2, page 324) Exercise 5

**Exercise 6.** (Grimaldi, 5. ed., Exercises 6.3, page 332) Exercise 7 b), c)

**Exercise 7.** Let  $\Sigma := \{a, b, c, d\}$  be an alphabet. Find regular languages corresponding to the following regular expressions. Note, if the set is infinite, then list the first ten elements.

a)  $a(b \vee c \vee d)a$

b)  $a^*b^*c^*$

c)  $a(bc)^*d$

**Exercise 8.** Let  $\Sigma := \{a, b, c, d\}$  be an alphabet. Find regular expressions that correspond to the following regular languages.

a)  $\{ab, ac, ad\}$

b)  $\{ab, ac, bb, bc\}$

c)  $\{a, ab, abb, abbb, abbb, \dots\}$

**Exercise 9.** Let  $\Sigma := \{a, b, c\}$  be an alphabet.

a) Give a regular expression for the language  $L_1 \subset A^*$  where all elements have exactly two  $b$ 's.

b) Give a regular expression for the language  $L_2 \subset A^*$  where all elements have exactly two  $b$ 's and two  $c$ 's.

c) Give a regular expression for the language  $L_3 \subset A^*$  where all elements have one or more  $a$ 's, followed by one or more  $b$ 's and then one or more  $c$ 's.

**Exercise 10.** a) Draw the state diagram  $D(M)$  of the automaton  $M$  with states  $S := \{s_0, s_1, s_2\}$ , accepting states  $Y := \{s_1\}$ , input alphabet  $I := \{a, b\}$ , described in the state table  $T(M)$  on top of the next page.

b) Write a regular expression for the language accepted by  $M$ .

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	$\nu$	
	$a$	$b$
$s_0$	$s_0$	$s_1$
$s_1$	$s_0$	$s_2$
$s_2$	$s_2$	$s_2$

**Exercise 11.** Find an automaton  $M$  that accepts the regular language given by the regular expression  $(a^*(ba)^*bb^*a)^*$ .

## 2. CLASSROOM SET 12

**Exercise 12.** (Grimaldi, 5. ed., Exercises 6.1, page 317) Exercise 18 (Proof of Thm. 6.1 in  $G$ 's book, i.e., Thms. IX. 11 of lecture 26)

**Exercise 13.** (Grimaldi, 5. ed., Exercises 6.1, page 317) Exercise 12 a), b), c)

**Exercise 14.** (Grimaldi, 5. ed., Exercises 6.2, page 324) Exercise 3

**Exercise 15.** (Grimaldi, 5. ed., Exercises 6.3, page 332) Exercise 7 a)

**Exercise 16.** Find regular languages corresponding to the following regular expressions. Note, if the set is infinite, then list the first ten elements.

a)  $(a \vee b)(c \vee d)$

b)  $(ab^*\lambda) \vee (cd)^*$

**Exercise 17.** Let  $\Sigma := \{a, b, c, d\}$  be an alphabet. Find regular expressions that correspond to the following regular languages.

a)  $\{ab, abab, ababab, abababab, \dots\}$

b)  $\{ab, abb, aab, aabb\}$

**Exercise 18.** Let  $\Sigma := \{a, b, c\}$  be an alphabet.

a) Give a regular expression for the language  $L_1 \subset A^*$  where all elements have two or more  $b$ 's.

b) Give a regular expression for the language  $L_2 \subset A^*$  where all elements begin and end with  $a$  and contain at least one  $b$  and one  $c$ .

**Exercise 19.** a) Draw the state diagram  $D(M)$  of the automaton  $M$  with states  $S := \{s_0, s_1, s_2\}$ , accepting states  $Y := \{s_0, s_2\}$ , input alphabet  $I := \{a, b\}$ , described in the state table  $T(M)$  on top of the next page.

b) Which of the following input words are accepted by  $M$ ?

1)  $abba$

2)  $aabbb$

3)  $babab$

	$\nu$	
	$a$	$b$
$s_0$	$s_1$	$s_0$
$s_1$	$s_2$	$s_0$
$s_2$	$s_2$	$s_1$

4)  $aaabbb$

5)  $bbaab$

**Exercise 20.** a) Draw the state diagram  $D(M)$  of the automaton  $M$  with states  $S := \{s_0, s_1, s_2\}$ , accepting states  $Y := \{s_0\}$ , input alphabet  $I := \{a, b\}$ , described in the state table  $T(M)$ :

	$\nu$	
	$a$	$b$
$s_0$	$s_0$	$s_1$
$s_1$	$s_0$	$s_2$
$s_2$	$s_2$	$s_2$

b) Write a regular expression for the language accepted by  $M$ .

**Exercise 21.** Find an automaton  $M$  that accepts the regular language given by the regular expression  $aa^*bb^*cc^*$ .