1 Evaluate the following improper integrals, or show that they diverge:

$$\int_0^1 \ln x \,\mathrm{d}x, \qquad \int_0^1 \frac{\mathrm{d}x}{\sqrt{x(1-x)}}, \qquad \int_0^{\pi/2} \tan x \,\mathrm{d}x,$$

$$\int_e^{\infty} \frac{\mathrm{d}x}{x\ln x}, \qquad \int_{-\infty}^{+\infty} \frac{x\mathrm{d}x}{1+x^4}, \qquad \int_{-\infty}^{+\infty} e^{-|x|}\mathrm{d}x.$$

2 Decide if the following improper integrals converge or diverge. Justify your answer.

$$\int_0^{\infty} \frac{x^2\mathrm{d}x}{x^5+1}, \quad \int_0^{\infty} \frac{\mathrm{d}x}{1+\sqrt{x}}, \quad \int_0^{\infty} e^{-x^3}\mathrm{d}x, \quad \int_{-1}^{1} \frac{e^x}{x+1}\mathrm{d}x\,.$$

3 Let

$$F : \left(0, \frac{\pi}{2}\right) \to \mathbb{R}, \qquad F(\theta) = \int_{\cos\theta}^{\sin\theta} \frac{\mathrm{d}x}{1-x^2}\,.$$

Find the critical points and the inflection points of F.

4 Is

$$\int_0^{\pi} \frac{\sin x}{x}\mathrm{d}x$$

an improper integral or a Riemann-integral?

5 Let $a < b$ and $f : [a,b] \to \mathbb{R}$ be a continuous function. Find the value of $t \in \mathbb{R}$ such that the integral

$$\int_a^b (f(x)-t)^2\mathrm{d}x$$

is minimized.

6 Find

$$\lim_{n\to\infty} n \int_{1}^{1+\frac{1}{n}} \frac{\cos(\pi t^2)}{t^2+1} \mathrm{d}t.$$

7 (Revision) If f is a differentiable and injective function defined on an open interval which contains $x = 1$, and f satisfies

$$f(1) = 3 \qquad \text{and} \qquad f'(1) = 2,$$

evaluate

$$(f^{-1})'(3).$$

8 (Revision) Let

$$f(x) = \begin{cases} 3x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0. \end{cases}$$

a) Show with an ε-δ argument that f is continuous at $x = 0$.

b) Is f differentiable at $x = 0$? (Prove or disprove.)

c) Decide if f is uniformly continuous on $[-1, 1]$.