Norges teknisk–naturvitenskapelige universitet
Institutt for matematiske fag

MA1101 Grunnkurs i analyse I
Høst 2020

Øving 11

1 Classify the following differential equations as "linear" or "non-linear" and in the linear case as "homogeneous" or "inhomogeneous".

a)
$$\frac{d^2y}{dx^2} + x = y.$$

b)
$$\cos x \frac{dx}{dt} + x \sin t = 0.$$

c)
$$x^2 y'' + \mathrm{e}^x y' = \frac{1}{y}.$$

2 Find the general solution of the differential equations

a)
$$y' - \frac{2y}{x} = x^2,$$

b)
$$y' + \frac{2y}{x} = \frac{1}{x^2},$$

c)
$$y' - \tan x \cdot y = 1.$$

d)
$$y' = \frac{x+y}{x-y},$$

e)
$$\frac{dx}{dt} = e^x \sin t,$$

f)
$$xy' = y + x\cos^2\left(\frac{y}{x}\right),$$

g)
$$y' = y^2(1-y).$$

3 Solve the initial value problems

a)

$$y' + 2xy = \mathrm{e}^{-x^2}, \qquad y(0) = 1$$

b)

$$y' = 2(y^2 + 1)x, \qquad y(0) = 1$$

c)

$$y' + 10y = 1, \qquad y\left(\frac{1}{10}\right) = \frac{1}{5}$$

4 Solve the following integral equations

a)

$$y(x) = 2 + \int_0^x \frac{t}{y(t)} dt$$

b)

$$y(x) = 3 + \int_0^x e^{-y(t)} \mathrm{d}t$$

5 Decide if the following improper integrals converge or diverge. Justify your answer.

a)

$$\int_2^\infty \frac{x\sqrt{x}}{x^2 - 1} \mathrm{d}x$$

b)

$$\int_1^\infty \frac{x + \sin x}{x^2 + \sin x} \mathrm{d}x$$

c)

$$\int_0^\infty \frac{\mathrm{d}x}{xe^x}$$

d)

$$\int_{-\pi/2}^{\pi/2} \frac{\mathrm{d}x}{\sin x}$$

6 Find the values of $p \in \mathbb{R}$ such that the integral

$$\int_0^1 \frac{\sin x}{x^p} \mathrm{d}x$$

converges.

7 Find a function which is:

a) bounded on $[-1, 1]$, but not continuous at the origin.

b) continuous at the origin, but not uniformly continuous on $[-1, 1]$.

c) uniformly continuous on $[-1, 1]$, but not differentiable at the origin

d) differentiable on $[-1, 1]$, but not continuously differentiable (ie. the derivative is not continuous) on the same set.

Give short but correct arguments for why the functions satisfy the required properties in each case.