Norges teknisk–naturvitenskapelige universitet
Institutt for matematiske fag

MA1101 Grunnkurs i analyse I
Autumn 2020

Øving 12

1 Let

$$A = \left\{ \frac{n-1}{n+1} : n = 1, 2, \dots \right\}.$$

Find $\max A, \min A, \sup A, \inf A$.

2 Let

$$f : \mathbb{R} \to \mathbb{R}, \qquad f(x) = \frac{x^3+1}{x^2+1}.$$

a) Find all asymptotes of the graph of f.

b) Find the second degree McLaurin polynomial of f.

c) Prove that the equation $f(x) = \frac{3}{2}$ has a unique solution in $[1, 2]$.

d) Find the area among the graph of f, the horizontal axis and the lines $x = 0$ and $x = 1$.

3 Find

$$\lim_{n\to\infty} \frac{1}{n^2} \sum_{k=1}^{2n} (-1)^k k(k+1).$$

4 Evaluate the following series.

a)

$$\sum_{n=1}^{\infty} \frac{1}{n^2+7n+12}$$

b)

$$\sum_{n=0}^{\infty} \frac{2^n}{n!}$$

5 Use the ε - δ definition to show that the function $f(x) = x^3 + 2$ is continuous at the point $x_0 = -1$.

6 Evaluate

$$\int_0^1 x \arcsin x \mathrm{d}x.$$

7 Solve the initial value problems

a)

$$\begin{cases} y' + (\cos x)y &= 2xe^{-\sin x} \\ y(\pi) &= 0 \end{cases}$$

b)

$$\begin{cases} x^2y' + y &= x^2e^{1/x} \\ y(1) &= 3e \end{cases}$$

8 Decide if the following improper integrals converge or diverge. Justify your answer.

a)

$$\int_{-1}^1 \frac{1}{x^2-1}\mathrm{d}x$$

b)

$$\int_1^\infty \frac{\cos^2 x}{x^4+1}\mathrm{d}x$$

9 Find

$$\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{2n}\right)$$