



- 1 Find the number of real roots of the polynomial

$$P(x) = x^5 - x + 1.$$

- 2 Evaluate the following series.

a)

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$$

b)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!}$$

- 3 Solve the initial value problems.

a)

$$\begin{cases} xy' - y = 1 \\ y(1) = 1 \end{cases}$$

b)

$$\begin{cases} y' = 2xe^{-y} \\ y(0) = 0 \end{cases}$$

- 4 Decide if the following improper integrals converge or diverge. Justify your answer.

a)

$$\int_e^{\infty} \frac{dx}{\ln x}$$

b)

$$\int_0^1 \frac{dx}{e^x - e^{-x}}$$

- 5 Let $A, B \subseteq \mathbb{R}$ be two non-empty bounded sets of real numbers such that $A \subseteq B$. Show that

$$\sup A \leq \sup B.$$