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This is the fifth exercise sheet. Observe the minimum requirement of 8/12 exercise sheets completed to pass the exam. Check your status in Blackboard.

2.6.17 Let $f(x) = \dfrac{1}{a+bx}\cdot$

a) Find $f^{(3)}(x)$.

b) Find a formula for $f^{(n)}(x)$ for $n \in \mathbb{N}$.
BONUS: Prove by induction that this expression is correct.

2.8.5 Show that

$$\tan x > x \quad \text{when} \quad 0 < x < \frac{\pi}{2}\cdot$$

A geometric argument is not considered a proof, only an analytic!

3 Find a continuous and differentiable real function f with $f(0) = f'(0) = 0$, which nevertheless is strictly increasing, that is,

$$f(x) > f(y) \quad \text{whenever} \quad x > y.$$

This shows that the property of the derivative having constant sign is not the same as strict monotony; it is a stronger property.

2.8.29 Formulate and prove Darboux's theorem.

2.9.10 Use implicit derivation to find the tangent of the following curve at the point (x, y) when

$$x^2y^3 - x^3y^2 = 12, \qquad (x, y) = (-1, 2).$$

6 Let $z = \tan x$.

a) Show that $1 + z^2 = \dfrac{1}{\cos^2 x}$.

b) Given that $\dfrac{dz}{dx} = \frac{1}{dx/dz}$ when both quotients are finite, show that

$$\frac{dx}{dz} = \frac{1}{1+z^2}\,.$$

You have just calculated the derivative of the function arctan.

4.3.3,5,7,34 Calculate

a) $\displaystyle\lim_{x\to 0} \frac{\sin(ax)}{\sin(bx)}$, for $a, b > 0$.

b) $\displaystyle\lim_{x\to 0} \frac{\arcsin x}{\arctan x}$

c) $\displaystyle\lim_{x\to 0^+} \frac{\sin^2 x}{\tan x - x}$

d) $\displaystyle\lim_{h\to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$, given that f is twice differentiable.

4.4.21 Find and classify all extreme and critical points of

$$f(x) = x^3(x-1)^2.$$

Sketch the graph of f.

2.8.32 Let $f : I \to \mathbb{R}$, where I is an interval. If f is twice differentiable and vanishes at at least three distinct points of I, prove that f'' vanishes at some point in I.

2.8.33 Let $f : [0,2] \to \mathbb{R}$ be a function. If f is twice differentiable in $(0,2)$ and $f(0) = f(1) = 0, f(2) = 1$, prove that

a) there exists $x_1 \in (0,2)$ such that $f'(x_1) = \dfrac{1}{2}$,

b) there exists $x_2 \in (0,2)$ such that $f''(x_2) > \dfrac{1}{2}$,

c) there exists $x_3 \in (0,2)$ such that $f'(x_3) = \dfrac{1}{7}$.