Norges teknisk–naturvitenskapelige universitet
Institutt for matematiske fag

Observe the minimum requirement of 8/12 solved exercise sheets to pass the exam. Check your status in Blackboard. See updated information for the midterm exam in the homepage.

1 Show that

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}\,.$$

Hint: $(k+1)^4 - k^4 = 4k^3 + 6k^2 + 4k + 1$.

2 Evaluate the following sums.

a)

$$\sum_{k=1}^{n}(2k-1)$$

b)

$$\sum_{k=1}^{n}(2k-1)^2$$

3 Assume $\sum_{k=1}^{\infty} a_k$ converges. Show that

$$\lim_{n\to\infty}(a_n + a_{n+1}) = 0.$$

4 Decide if the following series converge, and if they do find their values.

a)

$$\sum_{n=1}^{\infty}\frac{1}{n(n+2)}$$

b)

$$\sum_{n=1}^{\infty}\frac{1}{n^2+5n+6}$$

c)

$$\sum_{n=1}^{\infty}\frac{1}{2n-1}$$

d)

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$$

5 Decide whether the series

$$\sum_{n=1}^{\infty} \log\left(\frac{n+1}{n}\right)$$

converges or not.

6 **a)** Let $n \in \mathbb{N}$. Show that $\sum_{k=1}^{\infty} a_k$ converges if and only if $\sum_{k=n}^{\infty} a_k$ converges.

b) Assume $\sum_{k=1}^{\infty} a_k$ converges. By the previous question, for each $n = 1, 2, \ldots$ there exists R_n such that $R_n = \sum_{k=n}^{\infty} a_k$. Show that

$$\lim_{n\to\infty} R_n = 0.$$

7 Evaluate

$$\sum_{n=1}^{\infty} \frac{1}{2^{3n+1}}.$$