MA1101 Grunnkurs i analyse I
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Øving 9

1 Let $f : \mathbb{R} \to \mathbb{R}$ be a Riemann-integrable function with

$$\sup_{x\in\mathbb{R}} |f(x)| = B$$

for some real number B. Show with an $\varepsilon - \delta$ argument that the function $F : \mathbb{R} \to \mathbb{R}$ given by

$$F(x) = \int_0^x f(s)\,\mathrm{d}s$$

is continuous at every point $x_0 \in \mathbb{R}$.

2 Evaluate

$$\int_{\sqrt{3}/3}^{\sqrt{3}} \frac{e^{\arctan x}}{1+x^2}\mathrm{d}x, \quad \int_2^3 \frac{\mathrm{d}x}{x^2-5x+4}, \quad \int_1^3 \frac{\operatorname{sgn}(x-2)}{x^2}\mathrm{d}x, \quad \int_{-2\pi}^{2\pi} x^2\sin(x^5)\mathrm{d}x,$$

$$\int_{1/2}^1 \frac{\arcsin x}{x^2}\mathrm{d}x, \quad \int_1^2 \frac{\mathrm{d}x}{x^2\sqrt{9-x^2}}.$$

3 Evaluate the following improper integrals or show they diverge:

$$\int_2^\infty \frac{\mathrm{d}x}{(x-1)^3}, \quad \int_{-\infty}^{-1} \frac{\mathrm{d}x}{x^2+1}, \quad \int_{-1}^1 \frac{\mathrm{d}x}{(x+1)^{\frac{2}{3}}}, \quad \int_0^a \frac{\mathrm{d}x}{a^2-x^2}, \quad \int_0^1 \frac{\mathrm{d}x}{x\sqrt{1-x}}.$$

4 The following calculation contains an error:

$$\int_{-1}^{1} \frac{\mathrm{d}x}{x^2} = -\frac{1}{x}\bigg|_{-1}^{1} = -1 + \frac{1}{-1} = -2.$$

Find the mistake in the preceding calculation. Why can -2 not be a reasonable value of the integral?

5 Use the addition formulae for $\cos(x \pm y)$ to show that

$$\cos x \cos y = \frac{1}{2}\left(\cos(x-y) + \cos(x+y)\right).$$

Use this to show that

$$\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\mathrm{d}x = 0,$$

for all integers m, n with $m \neq n$.